A GENERAL ANALYTIC APPROACH TO ANALYSING NONSTATIONARY STOCHASTIC PROCESSES IN THERMODYNAMIC SYSTEMS. PART II: BASIC FEATURES OF THE CONSTITUTIVE EQUATION ANALYTIC SOLUTIONS

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Abstract. As prolongation of the Part I of the article, the further stages of the proposed approach development were conducted to provide the mandatory details of a transient process evolution in a thermodynamic system: linear analysis of stability for the analytic solutions of the constitutive nonlinear differential equation; delay time dependence of the autocorrelation function for each of two revealed modes of the analytic solutions; external factors effect on the energy carrier velocity. For the case of the crystal plastic deformation, the obtained results were interpreted in view of possible dislocation mechanisms in mono- and polycrystalline solids. The first mode of the transient development was grounded to be considered as corresponding to the single dislocation glide, where presumably original, non-interacting each other dislocations begin to glide sporadically at an arbitrary point of time and space in a crystal slip planes. The second mode of the transient development was associated with the multiple dislocation glide, which includes periodically developed secondary one, combined with numerous acts of original dislocation segments as the dislocation sources. A new equation reflecting time and an external factor dependence of energy carrier velocity in a thermodynamic system was deduced within the framework of the developed approach. The equation is in accordance with the known ones and provides a basis to specify the values of the integration constants of the developed approach. For the case of plastic crystal deformation, it was shown that the movement of only one dislocation segment triggers the whole crystal plastic flow.

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1. INTRODUCTION

As it was shown in Part I of the article, a transient process in a thermodynamic system may be well basically described by the analytic approach formulated in [1]. The proposed approach is founded on some commonly accepted, well theoretically and experimentally grounded assumptions. The relevant main mathematical relations were deduced from the assumptions including constitutive non-linear differential equation describing the time evolution of the introduced process parameter $s = (\tilde{\sigma}/\sigma)^n$. Two different modes $s_j = s_j\,(t)$, where $j = 1,\ 2$, of a transient process development in a thermodynamic system were revealed by analytical solving the equation. As an example of the transient process, the macroscopic plastic deformation of a Face-Centred Cubic (FCC) single phase polycrystal was considered due to the most detail relevant theoretical and experimental data available. The revealed differences of the transient modes were particularly attributed in this case to various types of dislocation barriers providing different dislocation glide resistance $\tilde{\sigma}_j$ or the barriers dissociation stress in a deformed crystal. This conclusion was confirmed by an evaluation of the dissociation stress for the dislocation barriers under the single and multiple slip conditions in Cu-single crystals. An important task of the investigation having to be conducted is to propose and elaborate the dislocation mechanisms corresponding to the revealed mathematical features of the analytic relations.

So, the aim of the Part II of the paper presented here is to conduct the following necessary investigation stages for the solutions of a nonlinear differential equation: linear stability analysis; evaluation of the autocorrelation levels for each of the modes, calculation of the average velocity of the energy carrier displacement within the thermodynamic system together with the physical interpretation of the results.

2. STABILITY OF THE ANALYTIC SOLUTIONS OF THE CONSTITUTIVE EQUATION

Stability evaluation for solutions of a differential equation is the inevitable stage of its full solving [2]. Regarding autonomous system (13) deduced in [1]

as a constitutive relation, the relevant mathematical analysis was conducted in [3]. Authors noticed the well-known impossibility of analytic solving the nonlinear differential equations in general form and the importance of the stability analysis for the cases when such solutions are possible.

As it was shown in Part I of the article [1], the analytic solutions of the constitutive equation (13) may be particularly associated with the corresponding crystal plastic deformation modes. So, an important stage of the properties investigation for the solutions is the evaluation of the stability for the transient development modes (14) and (15) [1] in a thermodynamic system under given external factors action. In accordance with the standard linear stability analysis procedure [2], we should consider the stationary states of the autonomous system (13) [1] in the phase space $\{\ddot{s}, \dot{s}, s\}$. However, before the analysis beginning, some features of the phase portrait of the system (13) [1], revealed as a result of its analytical solving, must be taken into account. It was particularly ascertained that the variable $\ddot{s}$ is not a function of the other two phase variables $\dot{s}, s$. Indeed, an expression for $\dddot{s}$ that flows from the definition of $\ddot{s}$ as a function of two variables is not equivalent to the main equation (13) [1]:

$$\dddot{s} \equiv \frac{d\ddot{s}}{dt} = \ddot{s}\left(\frac{\partial\, \ddot{s}}{\partial\, \dot{s}}\right) + \dot{s}\left(\frac{\partial\, \ddot{s}}{\partial\, s}\right) \neq \ddot{s}\,(3\dot{s}) + \dot{s}\left(-\dot{s}^2\right).$$

This result allows to consider the corresponding trajectories in the phase planes $\{\ddot{s}, s\}$,$\{\ddot{s}, \dot{s}\}$,$\{\dot{s}, s\}$as independent ones. Another feature of the portrait flows from an examination of self-consistency of the solutions of the system (13) [1] performed by the following mean. On the one hand, expressions for time dependencies of the first and second derivatives of the quantities $s_j\,(t)$ were obtained by direct differentiation of the equations (14) and (15) from [1]. On the other hand, formulae describing the same dependencies were found using the pair relations between the derivatives obtained as a result of the analytical solution of the system (13) [1]:

$$\dot{s}_j = F_1\,(s_j)\,, \tag{1}$$

$$\ddot{s}_j = F_2\,(\dot{s}_j)\,, \tag{2}$$

$$\ddot{s}_j = F_3\,(s_j)\,. \tag{3}$$

The comparison of the corresponding expressions found by the two different methods had shown, first at all, that the dependencies (3) do not satisfy the main equation (13) from [1]. Besides, it was ascertained that the dependencies (1) are valid within the time interval $0 < t < \infty$, meanwhile, the rest ones

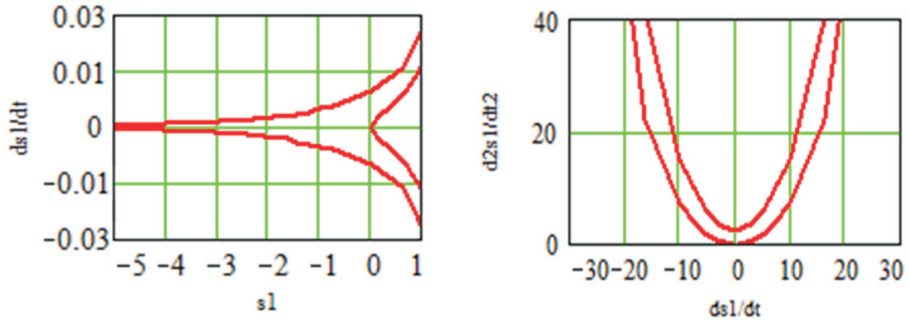


Fig. 1. Phase trajectories for the first group of the analytic solutions of the nonlinear differential equation (13), [1]

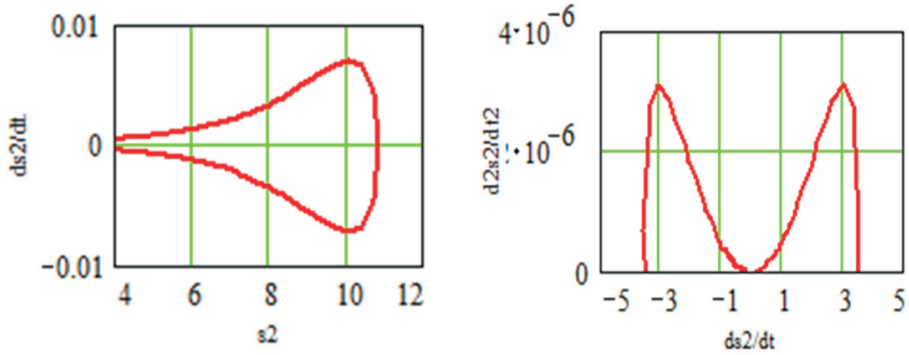


Fig. 2. Phase orbits for the second mode of the transient process development in a thermodynamic system defined by the nonlinear differential equation (13), [1]

described by the equation (2) are true only at $t > \hat{t}_j \left(1 + \sqrt{2}\right)$. Trajectories of image points in the phase planes $\{\ddot{s}, \dot{s}\}$ and $\{\dot{s}, s\}$, corresponding to the two revealed groups ($j = 1, 2$) of the system (13) [1] solutions, are shown in Figs 1 and 2. It should be emphasized a common feature of both the above phase portraits: their limit equal to zero at $t \to +\infty$ and $t \to -\infty$ indicates their "homoclinic" type [2]. It is well known that such phase orbits reflect a chaotic component in behaviour of a dynamic system and have been observed yet only in 3-dimensional phase space. Besides, as it follows from Fig. 3 shown in [1], the break in curve $s_1(t)$ at the point $t = \hat{t}_1 \left(1 + \sqrt{2}\right)$ corresponds to the explosion-like change of a thermodynamic system state during the first transient mode development. Therefore, the stability analysis for the first mode of the development should be performed separately for the two time

intervals. Because of restriction of the article size, we shall consider only the initial stage of the first deformation mode $0 < t < \widehat{t}_j \left(1 + \sqrt{2}\right)$. At that, as it follows from the above, only the trajectory $\dot{s}_1 = F_1(s_1)$ in the phase plane $\{\dot{s}, s\}$ described by the given above equation (1) should be considered.

In view of the results obtained, let us evaluate the stability of the stationary solutions of the system (13) [1] under the conditions of jointly changed phase variables. According to the standard procedure, the following pairs of differential equations for each mode of the macroplastic flow ($j = 1, 2$) should be analyzed:

$$\frac{d\dot{s}_j}{ds_j} = \frac{M_j(\dot{s}, s)}{N_j(\dot{s}, s)}, \tag{4.1}$$

$$\frac{d\ddot{s}_j}{d\dot{s}_j} = \frac{Q_j(\ddot{s}, \dot{s})}{P_j(\ddot{s}, \dot{s})}. \tag{4.2}$$

Because of restrictions of the article size, the full expressions for the functions on the right-hand side of these equations are not written here, but they may be easily obtained from the relations (14) and (15) [1]. The analysis of the equations (4.1) and (4.2) showed that only two pairs of the specific points with coordinates $\dot{s}_{js} = 0$, $s_{js} = \ln(A_j/4)$ and $\dot{s}_{js} = 0$, $\ddot{s}_{js} = -1/2\widehat{t}_j^{\,2}$ exist for each above mode of the transient process development in a thermodynamic system.

For the case of the first mode of the transient development, it was revealed the unique equilibrium point $\dot{s}_{1s} = 0$, $s_{1s} = \ln(A_1/4)$ within the time interval $0 < t < \widehat{t}_1 \left(1 + \sqrt{2}\right)$ where the parameter $s_1(t)$ varies continuously (See Fig. 2, curve 1 in [1]). This unstable point is known as the "saddle" [2]. At that, the equilibrium value of the phase variable $\dot{s}_1(t)$, i.e. $\dot{s}_{1s} = 0$ is reached at $t = \hat{t}_1$, but the corresponding value $s_{1s} = \ln(A_1/4)$ of the other variable $s_1(t)$ cannot be reached at any time.

Thus, in the specific case of the crystal deformation, the above results show that during the first its mode, a gliding active dislocation segment passes over the unstable condition reached by the time $t = \hat{t}_1$ and corresponding to the top of the "saddle". Besides, as it flows from existence of the equivalent solutions of the type (16) [1] for the considered first deformation mode, the other active dislocation segments, as already existing as just generated ones, undergo the same evolution of their states at later stages of the crystal deformation. Taking into account that such segments do not interact with each other, which follows from the basic definition used to derive the expression (10) [1], we may consider the constant t_{01} from the relation (16) in [1] as an arbitrary parameter and

the new dislocation segments as appearing in random points of both time and space in the slip planes.

It was ascertained also that the specific points of the second mode of the transient process development are the "centers" [2]. Consequently, the deviations of the phase variables $\dot{s}_2(t)$ and $s_2(t)$ out of their equilibrium values $\dot{s}_{2s} = 0$ and $s_{2s} = \ln A_2/4$, along the corresponding trajectory in the phase plane $\{\dot{s}, s\}$, must be defined by the relations:

$$\delta\ \dot{s}_2 = \delta\ \dot{s}_{20} \cdot \cos(\omega_1\ t + \varphi), \quad \delta\ s_2 = \delta\ s_{20} \cdot \cos(\omega_1\ t + \varphi), \tag{5}$$

where $\delta\ \dot{s}_{20}$, $\delta\ s_{20}$ are maximum deviations of the corresponding phase variables, ω_1 is the proper circle frequency of the deviations, φ is an initial phase of the oscillations. It was found that:

$$\omega_1 = 8\sqrt{2}\Big/\hat{t}_2.$$

On the other hand, as it follows from the equations (15) in [1] and (1) in the current article, the same phase variables have to vary non-periodically with time.

The analogous combination of the periodic and non-periodic behaviour of the phase variables $\dot{s}_2(t)$ and $\ddot{s}_2(t)$ with time also takes place in vicinities of the equilibrium point $\dot{s}_{2s} = 0$, $\ddot{s}_{2s} = -1/2\,\widehat{t}^{\,2}_2$. At that, these periodic variations are described by the relations similar to (5), but characterized by the less proper circle frequency:

$$\omega_2 = \sqrt{2}\Big/\widehat{t}_2.$$

In order to reveal the details of the just described joint development of the periodic and non-periodic variations of the phase variables for the second mode of the transient process development in a thermodynamic system, let us take into account the following. It is easy to show that the relations of the type (5) obtained by solving the first order approximation system are not solutions of the main equation (13) [1]. On the other hand, as it was shown in [1], there is a set of the equivalent solutions (16) of the equation (13) for each of the two above transient development modes and corresponding phase variables. Consequently, the above oscillations of the variables $\dot{s}_2(t)$ and $s_2(t)$ should be associated with the periodic realization of such solutions having the form:

$$s_2(t - t_{02}) = \ln \frac{A_2}{2 + \left(\frac{t - t_{02}}{\widehat{t}_2} - 1\right)^2}. \tag{6}$$

Graphic illustration of the mentioned above combination of the periodic and non-periodic behaviour of the variables $\dot{s}_2(t)$ and $s_2(t)$ is shown in Fig. 3. Here we define $t_{02} = nT_1$ where $T_1 = 2\pi/\omega_1 \approx 0.56 \cdot \widehat{t}_2$ is the period of the quantity $s_2(t)$ oscillations and $n = 0, 1, 2, 3, \ldots$ is a sequentia number of the dependence in the series of possible equivalent ones. As it follows from the plots, after beginning the deformation at $t = 0$, the value of the quantity $s_2(t)$ varies with time along the first left curve ($n = 0$) until $t \approx 0.56 \cdot \widehat{t}_2$, where the new curve ($n = 1$) is realized. At that instant, the affix of the initial curve (for which $n = 0$) can return in its starting-point corresponding to the beginning of the curve with $n = 1$. At the same time, the affix is to continue its motion along the initial curve ($n = 0$) according to the relation (15) [1]. Later on, the just described processes are repeated for the each following curve $n = 2, 3, \ldots$, providing the combination of the periodic and non-periodic variations of $s_2(t)$. As it follows from the use of the linear analysis procedure, the above oscillations of $s_2(t)$ have to be terminated when the quantity value defined by the relation (15) [1], i.e. the affix of the curve for which $n = 0$ moves away from the equilibrium point: until the point $s_{2s} = \ln(A_2/4)$ will be reached by the time $t = \hat{t}_2\left(1 + \sqrt{2}\right)$. The results obtained above show that under the conditions $t > \widehat{t}_2\left(1 + \sqrt{2}\right)$, the new process of the quantity $s_2(t)$ oscillations characterized by the period $T_2 = 2\pi/\omega_2 \approx 4.44 \cdot \widehat{t}_2$ should develop further due to the periodic variations of the quantities $\dot{s}_2(t)$ and $\ddot{s}_2(t)$. Some curves corresponding to the just mentioned processes are also shown in Fig. 3.

The results obtained agree with the model assumption used to derive the constitutive equation (13) [1], according to which any transient in a thermodynamic system is a third order one [4]. In the considered specific case of the crystal macro-plastic deformation in the present article, it means that a loaded crystal during the deformation process arrives many times to the following states: (i) the beginning of the macroplastic flow; (ii) deformation hardening; and (iii) termination of the plastic flow. It may be particularly interpreted in terms of involving new crystal volumes, such as the grains in polycrystals, in the plastic deformation process. As it was shown above, the time dependence $s_2(t)$ characterizes the evolution of state not only of a whole loaded crystal, but of an active dislocation segment initiating the crystal deformation as well. At that, the curves shown in Fig. 3 have to be associated with the identical parts of the same just mentioned segment. On the other hand, the parts should be considered as new active dislocation segments, each of them is characterized by its own time dependence $s_2(t - nT_k)$ where (see

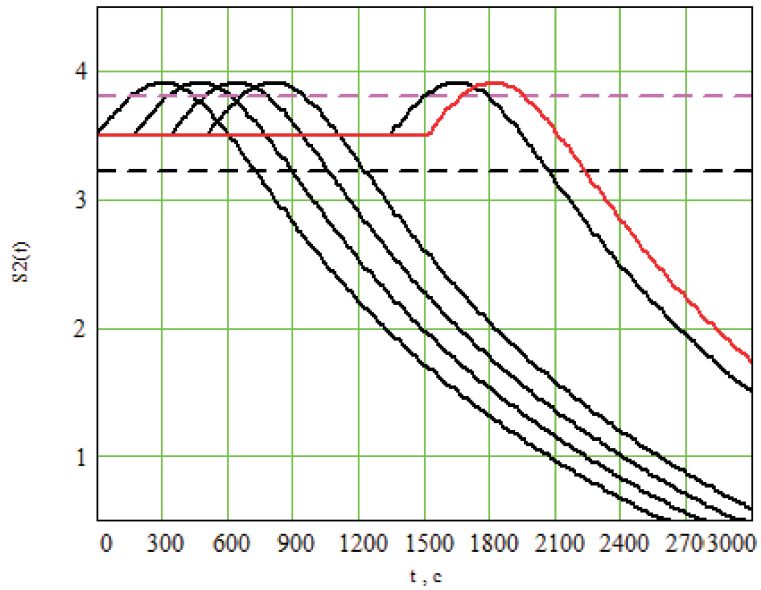
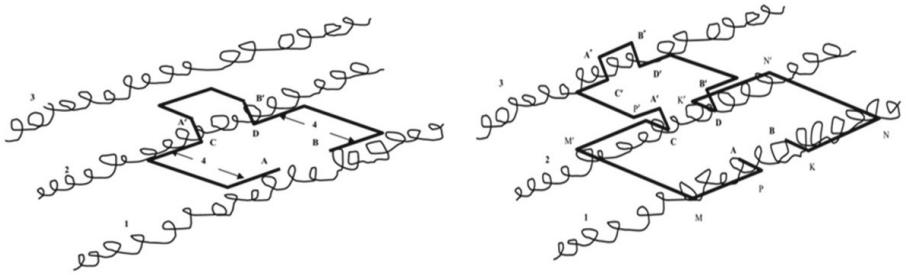


Fig. 3. Series of time dependencies of the variable $s_2\,(t - t_{02})$ at various values of the constant t_{02}

above) $k = 1,\ 2$. At that, some of the new segments appear in planes of a crystal predominant slip system with the period $T_1 \approx 0.56 \cdot \hat{t}_2$ only on initial stages of the deformation, while the others with the period $T_2 \approx 4.44 \cdot \hat{t}_2$ – during the whole deformation process. As it follows from Fig. 3, the beginning of the deformation ($t = 0$) is controlled by the long distance glide of only one of the pre-existing dislocation segments. By the time $t \approx 0.56 \cdot \hat{t}_2$, this moving segment arrives at the state at which beginning of the glide of a new, daughter, dislocation segment is possible. A daughter active segment of a new type is also generated by the same gliding parent dislocation by the time $t \approx 4.44 \cdot \widehat{t}_2$. On the other hand, the parent segment continues to glide, changing its state according to the non-periodic dependence (15) [1]. These results are in accordance with the commonly accepted notions about dislocation as a linear lattice defect, the various parts of which can be in different states simultaneously, as well as about the multiplication of dislocations during their glide, particularly by the multiple cross slip mechanism.

Besides, in accordance with the dislocation theory [5], the above results show existence of two possible processes of the multiplication of gliding dislocations under the jerky glide conditions. The just described details of the predicted periodic dislocation processes correspond to the multiple cross slip

of a gliding dislocation (the process with $T_1 \approx 0.56 \cdot \hat{t}_2$) in combination with its operation as a Frank-Reed source in original slip plane (the process with $T_2 \approx 4.44 \cdot \hat{t}_2$). Thus, it may be concluded that the second deformation mode is accompanied by the multiplication of gliding dislocations according to a mechanism involving the multiple cross slip. Since the cross slip is a particular case of the multiple one [6], the latter is regarded as a controlling dislocation mechanism for the second mode of the macroplastic crystal deformation. The described above progression of moving dislocation states during the plastic crystal deformation development may be illustrated by the schemes shown in Fig. 4.



(a) (b)

Fig. 4. Schematic illustration of the dislocation glide through the dislocation tangles (1, 2, 3, ...) formed on the crystal microplastic deformation stage due to the prevailing dislocation glide in a predominant slip system and restricted one in secondary slip planes: (a) beginning the tangles bypassing by the multiple cross slip of an original dislocation ABCD (the process with $T_1 \approx 0.56 \cdot \hat{t}_2$); (b) the tangles overcoming in a plane of an initial slip system by the operating original dislocation as a Frank-Reed source (the process with $T_2 \approx 4.44 \cdot \hat{t}_2$) (dislocation configuration PMM′P′K′N′NK)

The above results also show that the first deformation mode is not accompanied by periodic variations of the deformed crystal or the gliding dislocation state. Only sporadic appearance of new gliding active dislocation segments, independent of previous gliding ones, is allowed. It implies that neither multiple cross slip of a gliding active dislocation segment nor its operation as a Frank-Reed source in an original slip plane is possible during the first mode of the crystal deformation. A dislocation mechanism that fulfils the described conditions is the single slip [7–8], during of which the thermally activated dissociation of the dislocation barriers is possible, providing the sporadic glide of the original mobile crystal dislocations.

These conclusions are in accordance with the results well known for single crystals where the regions of preferential development of the single and multiple slip are directly observed [7–8]. They are also confirmed by the experimental data for polycrystalline metals and coincide with the theoretical notions successfully used to describe the deformation behaviour of polycrystals [7].

3. AUTOCORRELATIONS FOR THE DIFFERENT MODES OF A TRANSIENT DEVELOPMENT IN A THERMODYNAMIC SYSTEM

Additional information about features of the two revealed modes of a transient development in a thermodynamic system may be obtained by specifying Autocorrelation Function (ACF) for each of the modes. According to the known definition [4], Autocorrelation (AC) characterizes the existence of a link between values of the same stochastic process index at different points of observation time. In other words, AC reflects a similarity, repeatability or predictability of index values during a stochastic process development. As the quantitative characteristic of AC is ACF, for the most general ergodic transient processes considered within the current article, ACF ($C_j(\tau)$) is specified by the relation, which was used for each of the modes ($j = 1,\ 2$) [4]:

$$C_j(\tau) = \frac{1}{T}\int_0^T s_j(t)\, s_j(t+\tau)\, dt,$$

where τ is a time delay, T is a time interval of observation under a thermodynamic system.

Computer calculations show that the dependence $C_1(\tau)$ for the first mode of the transient development may be of two different types depending on the corresponding model constants values, namely: to be the delay time independent at $A_1 \gg 2$, $\widehat{t}_1 > 0_1$ and to have the non-monotonous time changes (see curve 1 in Fig. 5). As it follows from the ACF definition [4], the above first type of the dependence $C_1(\tau)$ corresponds to a relatively high and steady level of the predictability for the first mode of the transient development or, specifically, the crystal macroplastic deformation. As to the above second possible type of the dependence $C_1(\tau)$, it should be interpreted as corresponding to an increase of the transient indices predictability at the beginning stage of its development but – rising chaotic component of the index changes on the finishing stages of the transient process. These conclusions are in agreement with

the well known data about conserving (unchanging) and possible partial ordering the dislocation configurations during initial stages of the mono-crystal plastic flow [8]. Meantime, the disordered (tangled) dislocation configurations appearance at the finishing stages of a crystal plastic flow preceding a crystal fracture is the widely known phenomenon.

Regarding the second possible mode of the transient development, two possible types of the $C_2(\tau)$ dependence were revealed at various values of the model constants: monotonously increasing and decreasing dependence (see Fig. 6 curve 1 and 2, respectively). As it will be particularly shown in the Part III of the article, in the specific case of a crystal plastic deformation, a rising value of the A_2 constant from $A_2 = 10^5$ to $A_2 = 10^7$ at a constant level of the other model parameter $\widehat{t}_2 \approx 30$ s enlarges the deformed crystal elongation before fracture (finishing stage of the plastic flow) at the equal stress. Proceeding from this, it should be expected the intensive formation and further development of the dislocation substructure in a deformed crystal during the high strain hardening plastic deformation, which is observed at lesser A_2 values. Meantime, it is in contrast with the less hardening one observed at bigger A_2 values. Taking into account the deformed crystal temperature rise during a plastic deformation process and a larger time that the mobile dislocations have to spend for bypassing the obstacles by a cross slip during the less strain

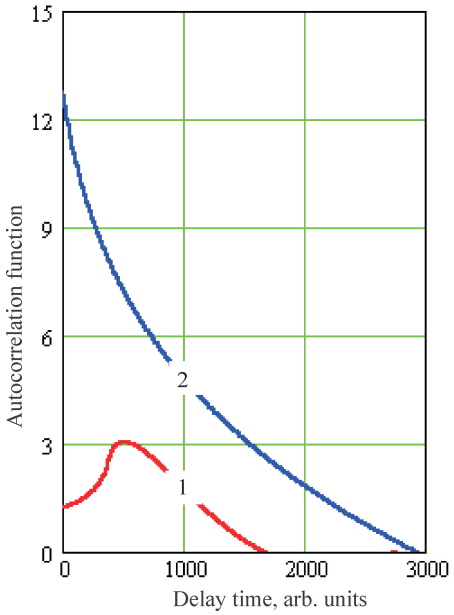


Fig. 5. Dependencies of the time autocorrelation functions $C_1(\tau) \cdot 10$ and $C_2(\tau)$ on delay time τ for the first (curve 1) and second (curve 2) mode of a transient development in a thermodynamic system, respectively, at the values of the model constants $A_1 = 2.02$, $\widehat{t}_1 = 1300$ s; $A_2 = 10^{4.2}$, $\widehat{t}_2 = 30$ s

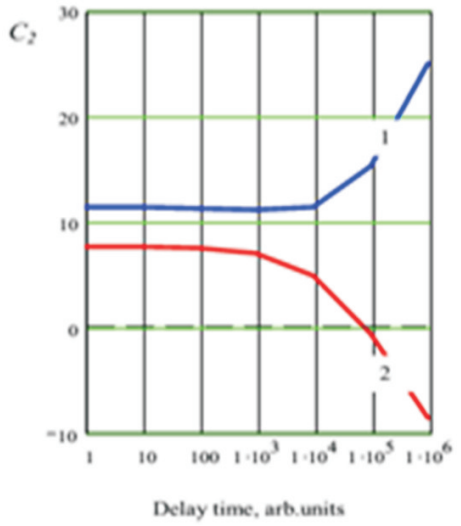


Fig. 6. Dependencies of the time autocorrelation function C_2 on delay time τ for the second mode of a transient development at the values of the model constants $A_2 = 10^5$, $\widehat{t}_2 = 30$ s, (curve1); $A_2 = 10^7$, $\widehat{t}_2 = 30$ s (curve 2)

hardened crystal plastic flow (under the action of less intensively growing internal stresses acting on a dislocation), the later deformation variant leads to higher thermal disordering the crystal dislocation substructure, probably due to the jogs formation. These results are also in agreement with the known data about formation of chaotic dislocation substructures in polycrystalline metals being in the fracture preceding states [8].

4. STRESS DEPENDENCE OF THE AVERAGE ENERGY CARRIER VELOCITY

As transient stochastic processes in a thermodynamic system transfer energy in general case, an important parameter of them is an average velocity of an energy carrier movement. For the considered specific case of the macroplastic deformation flow, the energy carriers are associated with mobile dislocations that redefine the velocity as an average velocity V_d of a mobile crystal dislocation.

Based on the approach developed in the article and according to [9–10], one can write:

$$V_d(s) = V_c \cdot \ P_m(s) + 0 \cdot P_{im}(s), \tag{7}$$

where V_c is a steady-state dislocation velocity which is equal to speed of sound in a metal crystal; $P_m(s)$ and $P_{im}(s)$ are the probabilities to find a mobile crystal dislocation in the moving and immobilized condition, respectively.

In order to determine the probability $P_m(s)$ let us specify the implication of expression (11) from [1], defining the probability $P_{II}(s)$. For that purpose, let us rewrite equation (11) from [1] in the form:

$$P_{II}(s) = (1-s) + \tfrac{1}{2!}s^2\left(1-\tfrac{1}{3}s\right) + \tfrac{1}{4!}s^4\left(1-\tfrac{1}{5}s\right) + \cdots$$

$$P_{II}(s) = (1-s) + \sum_{k=1} \tfrac{1}{2k!}s^{2k}\left[1-\tfrac{1}{(2k+1)}s\right]. \tag{8}$$

According to the definition (9) from [1], the first term of this expression represents the probability to find a driving active dislocation segment in the state of the thermally activated long-distance glide $(1-s) = p_{II}$. Any active dislocation segment, involved in a stable dislocation barrier with or without a dislocation pile-up in front of it, has to be taken into consideration due to the pile-up relaxation effect [5]. It follows from the probability theory that the first multiplier under the mark of the sum in equation (8), $1/2k!^{S^{2k}}$ is the probability of the joint immobilization of $2k$ active dislocation segments. The second multiplier describes the probability to find an active dislocation segment, chosen arbitrarily among the $(2k+1)$ equivalent segments, in the state of the long-distance glide. This probability depends on the number $2k$ of segments in the immobilized dislocation group. Furthermore, equation (8) shows that the whole crystal deformation is only possible as a result of the combination of long-distance glide of one of the $(2k+1)$ dislocation segments and immobilization of the other $2k$ dislocation group. Thus, the quantity $[1-s/(2k+1)]$ can be considered as the conditional probability to find one of the $(2k+1)$ equivalent active dislocation segments in the state of the long-distance glide while the other group of $2k$ dislocations is immobilized. The groups of $2k$ and $(2k+1)$ dislocations can therefore be interpreted as dislocation configurations formed as a result of a pile-up of active dislocation segments in front of some dislocation barriers. Taking the inevitable internal stress relaxation into account, the relaxed ("head") and non-relaxed ("tail") parts of a dislocation pile-up formed in front of a dislocation barrier are associated with $(2k+1)$ and $2k$ dislocation groups, respectively. Thus, equation (8) shows that the macroplastic crystal deformation is controlled by the long-distance glide of only one of the active crystal dislocation segments which is either the component of any stable dislocation barrier or involved in the relaxed part of a dislocation pile-up formed in front of such a barrier. Proceeding from the above, one can write $P_{II}(s) \equiv P_m(s)$, and finally obtain, substituting relation (11) from [1] into equation (7):

$$V_d(s) = V_c \cdot \exp\left[-\left(\frac{\tilde{\sigma}}{\sigma}\right)^n\right]. \tag{9}$$

It should be noted that the deduced formula transforms into the well-known empirical rate law for dislocations [9] in a particular case $n = 1$. The result means that the formula (9) corresponds to more general case to be considered, because according to the developed approach, $n = n(t)$ has to be a non-negative parameter, associated with a number of dislocation degrees of freedom [1]. Besides, as it was shown in the Part I and will be demonstrated in the Part III of the article, the formula (9) leads, respectively, to the well physically grounded relations (10) and (11) given in [1], as well as to true stress–true strain tensile diagrams, which are in a good agreement with the well-known dislocation mechanisms and the experimental data for the polycrystal macroplastic deformation. Meantime, the existing rate law rather well describes numerous experimental results for different rate processes [9–10], however only at high stresses. So, the relevant additional investigations should be conducted and only basic attributes of the time dependencies for the ratio $V_d(s)/V_c$ are outlined below for the revealed deformation modes.

As it follows from [1], the above time dependencies are described by the relations (14) and (15). Hence, considering Fig. 2, curve 3 [1], one can particularly conclude that in the course of the first deformation mode an average velocity of a gliding active dislocation segment can reach high values $V_d(s_1) \to V_c$ by the time $t = \widehat{t}_1$. This conclusion is in accordance with the experimental data demonstrating high levels of both the dislocation velocity ($\sim 2 \cdot 10^3$cm/s) within slip bands and the local strain rate in shear bands [11], e.g. after the beginning of the macroplastic polycrystal flow. It also agrees with the general kind of behaviour of gliding dislocations in an individual slip plane [9, 11].

For the second deformation mode, Fig. 2, curve 4 [1] and equation (9) show that an average dislocation velocity increases considerably $V_d(s_2) \to V_c$ by the time $t = \hat{t}_2 \cdot \left(1 + \sqrt{A_2 - 2}\right)$. This result is in a qualitative accordance with the experimental data showing a significant increase of the local crystal strain rate after the onset of the neck formation, i.e. on the final stages of the deformation [10]. Using equations (14), (15) from [1] and equation (9), it is also possible to specify the physical meaning of the model constants A_j and to obtain relations defining their values for the above modes of the macroplastic crystal flow:

$$A_1 = \frac{V_c}{V_{01}}; \quad A_2 = 3 \cdot \frac{V_c}{V_{02}}, \tag{10}$$

where V_{0j} is an average dislocation velocity at the onset of the crystal macroplastic deformation at the time $t = 0$. So, the additional investigations should be focused on specifying features of the dislocation velocity time dependencies at various mechanisms of a FCC crystals deformation.

5. CONCLUSION

1. In continuation of the Part I of the article where the analytic approach to investigating a transient process evolution in a thermodynamic system was formulated, the necessary stages of further approach development were conducted: linear stability analysis for the analytic solutions of the deduced nonlinear differential equation defining constitutive properties of the transient processes; autocorrelation levels evaluation for each of two revealed types of the analytic solutions; external factors effect on the energy carrier velocity.
2. Some new, unexpected features of the phase portrait of the constitutive nonlinear differential equation in the phase space $\{\ddot{s}, \dot{s}, s\}$ were revealed due to obtaining the analytical solution: the second derivative of the introduced in [1] parameter $s\,(t)$ is not a function of both the pair of variables $\dot{s}, s$ and the single variable s, which was explained by chaotic character of the transient evolution and confirmed by the "homoclinic" form of the calculated phase orbits.
3. For the case of the crystal plastic deformation, the obtained results of the linear stability and autocorrelation analysis were interpreted based on their ability to reflect the features of possible dislocation mechanisms in mono- and polycrystals.
4. The first mode of the transient development was attributed to the single dislocation glide, where at the beginning deformation stage, presumably original, preexisting, non-interacting each other dislocations begin to glide sporadically at arbitrary points of time and space in a crystal. This single glide stage is characterized by conserving the indices ordering levels, which decrease up to zero at the finishing deformation stage – crystal fracture.
5. The second mode of the transient development was associated with the multiple dislocation glide, which includes multiple (periodic) secondary one combined with numerous actions of original dislocation segments as the Frank-Reed sources. All the processes development leads to disordering the whole deformation phenomenon indices values.
6. A new equation for time and external factor dependencies of energy carrier velocity in a thermodynamic system was deduced from the constitutive equation of the developed approach. For the case of plastic crystal deformation, the physical probabilistic interpretation of the equation showed the movement of only one dislocation segment as a prerequisite for the development of the whole crystal plastic deformation.
7. The equation for the average velocity of a moving dislocation in a loaded crystal is in accordance with the known empirical relation, experimental

data, and provides a basis to specify the values of the constants of the developed approach.

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