

# DEFORMATION CHECKS ON THE SHAFTS OF BIG BAND SAW MACHINES – CASE A

BOYCHO MARINOV

*Institute of Mechanics, Bulgarian Academy of Sciences,  
“Acad. G. Bonchev” Str., Block 4, 1113 Sofia, Bulgaria,  
E-mails: boicho\_marinoff@yahoo.co.uk, boicho\_marinoff@imbm.bas.bg*

**Abstract.** In this study, deformation checks on upper shaft of big band saw machines have been made. For this purpose, the maximum deflections in the most endangered cross sections of the shaft are calculated. The biggest values are compared with the admissible values for the respective material. The normal operation of the big band saw machines is guaranteed if the actual values of the deflections are equal or less than the admissible values. The proposed solutions for the most endangered cross-section of the upper shaft allow to carry out deformation checks in the design of new band saw machines.

*Keywords:* deformation checks, deflections, spatial deformations, band saw machine, shafts, cross section, optimization procedures.

## 1. INTRODUCTION

Big band saw machines form a specific class of woodworking machines. They are used for sawing different types of wooden material. In order to saw logs, prisms and more, leading wheels with big diameters need to be used. Large static and dynamic loads arise in operating mode. These loads cause large deformations in the upper shaft as a result of which the shaft can be destroyed and the band saw machine to stop working. In this work, to avoid this unfavorable case, an algorithm for carrying out deformation checks of the most endangered cross sections is suggested. For this reason, expressions describing the full deflections of the upper shaft can be obtained. These expressions render an account the static and dynamic deformations of the shaft in two mutually perpendicular planes. Dependencies for their calculation were obtained by the author in previous articles and book [1, 2, 3]. The next step is to calculate the actual values of the deflections in the most endangered cross sections. To perform a deformation check of the shaft, the maximum value of the deflection must be determined. Last step is to perform the actual deformation check. In this case, the absolute or relative deflections in

---

DOI: 10.7546/EngSci.LIX.24.04.05

the most endangered cross sections of the upper shaft are used. These values of the deflections must be compared with the admissible absolute or relative deflections. Normal operation of the band saw machine is ensured if the real deflections are smaller or equal to the limit values established in the technical literature [4–8]. The proposed theory can be used to perform deformation checks in basic links of other classes of woodworking machines to ensure their reliable and safe work in the operating mode.

## 2. EXPOSE

### 2.1. Scheme of the cutting mechanism

The leading wheel of the big band saw machines can be mounted on the upper shaft in two ways. These two cases are written as case (a) and case (b) and are explained below:

case (a)– leading wheel is mounted at the end of the shaft,

case (b) – leading wheel is mounted between the two bearing supports.

The scheme of the cutting mechanism is shown in Fig. 1 [2, 3, 9]. In this section, we obtain expressions for performing a deformation check on the upper shaft when the leading wheel is mounted at the end of the shaft, i.e., case (a).

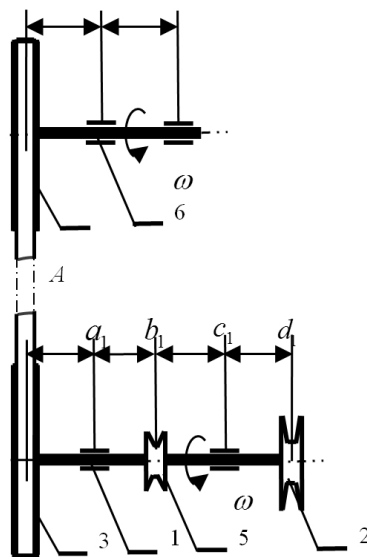
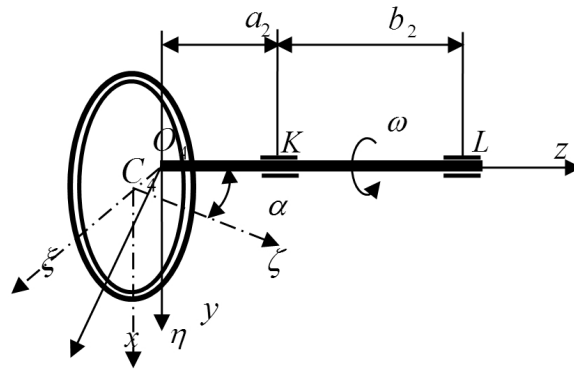


Fig. 1. Cutting mechanism

The following symbols are defined: 1 – main shaft, 2 and 5 – belt pulleys, 3 and 4 – leading wheels,  $A$  – band saw blade, and 6 – upper shaft.

## 2.2. Dynamic model



**Fig. 2.** Dynamic model

The dynamic model is shown in Fig. 2. The following coordinate systems are used: fixed coordinate system  $O_4xyz$ , and coordinate system  $C_4\xi\eta\zeta$  whose axes are principal axes of inertia. The linear and angular deviations  $e$  and  $\alpha$  are also shown in the figure as  $e = O_4C_4$  and  $\alpha$  is the angle between the axes  $\zeta$  and  $z$ .

## 2.3. Deformation checks-analytical solution

The upper shaft consists of two parts –  $O_4K$  and  $KL$ . Deformation checks can be performed analytically and numerically by using the following algorithm [10]:

- expressions describing the static and dynamic deformations of the shaft in two mutually perpendicular planes are used;
- the full deformations for the relevant axes are determined;
- the full deflection of the shaft for each part is calculated from known dependencies;
- the endangered cross-section of the shaft is determined. This is the section where the full deflection is the biggest. We denote this deflection with  $f_{\max}$ ;
- the calculated value of  $f_{\max}$  is used for deformation check of the shaft.

This algorithm is applied to each part.

I. Part  $O_4K$   $0 \leq z \leq a_2$ .

We write the expressions for determining the static and dynamic deformations of the shaft in two mutually perpendicular planes, taking into account the fact that the constants of integration in the expressions for dynamic deformations are equal for both planes.

$$\begin{aligned}
y_{O_4K}^{st}(z) &= \frac{1}{EJ} \left[ \frac{(G_4^e + 2F_4^e)}{6} z^3 + \frac{M_{G_4}^e}{2} z^2 + K_1 z + L_1 \right], \\
x_{O_4K}^{st}(z) &= \frac{1}{EJ} \left[ \frac{M_{R_4y}^{st}}{2} z^2 + K_3 z + L_3 \right], \\
y_{O_4K}^{dyn}(z, t) &= (\bar{K}_1 \cos \omega_0 z + \bar{L}_1 \sin \omega_0 z + \bar{M}_1 \operatorname{ch} \omega_0 z + \bar{N}_1 \operatorname{sh} \omega_0 z) \sin \omega t, \\
x_{O_4K}^{dyn}(z, t) &= (\bar{K}_3 \cos \omega_0 z + \bar{L}_3 \sin \omega_0 z + \bar{M}_3 \operatorname{ch} \omega_0 z + \bar{N}_3 \operatorname{sh} \omega_0 z) \cos \omega t.
\end{aligned} \tag{1}$$

The full deformations for the relevant axes are written below.

$$\begin{aligned}
y_{O_4K\Sigma}(z, t) &= y_{O_4K}^{st}(z) + y_{O_4K}^{dyn}(z, t) \\
x_{O_4K\Sigma}(z, t) &= x_{O_4K}^{st}(z) + x_{O_4K}^{dyn}(z, t)
\end{aligned} \tag{2}$$

The full deflection of the shaft is calculated by the next dependence.

$$f_{O_4K}(z, t) = \sqrt{(y_{O_4K\Sigma}(z, t))^2 + (x_{O_4K\Sigma}(z, t))^2}. \tag{3}$$

We transform the expression (3) and obtain the dependence for the full deflection of the upper shaft in the following form:

$$f_{O_4K}(z, t) = \sqrt{\operatorname{var} O_4K + 2(\bar{K}_1 \cos \omega_0 z + \bar{L}_1 \sin \omega_0 z + \bar{M}_1 \operatorname{ch} \omega_0 z + \bar{N}_1 \operatorname{sh} \omega_0 z) \cdot [y_{O_4K}^{st}(z) \sin \omega t + x_{O_4K}^{st}(z) \cos \omega t]}, \tag{4}$$

where

$$\begin{aligned}
\operatorname{var} O_4K &= (y_{O_4K}^{st}(z))^2 + (x_{O_4K}^{st}(z))^2 \\
&\quad + (\bar{K}_1 \cos \omega_0 z + \bar{L}_1 \sin \omega_0 z + \bar{M}_1 \operatorname{ch} \omega_0 z + \bar{N}_1 \operatorname{sh} \omega_0 z).
\end{aligned}$$

II. Part  $KL$   $a_2 \leq z \leq a_2 + b_2$ .

We apply the algorithm written above for part  $O_4K$  static and dynamic deformations of the upper shaft in two mutually perpendicular planes

$$\begin{aligned}
y_{KL}^{st}(z) &= \frac{1}{EJ} \left[ \frac{(G_4^e + 2F_4^e + K_y^{st})}{6} z^3 + \frac{(M_{G_4x}^e - K_y^{st} a_2)}{2} z^2 + K_2 Z + L_2 \right], \\
x_{KL}^{st}(z) &= \frac{1}{EJ} \left[ \frac{K_x^{st}}{6} z^3 + \frac{(M_{R_4y}^{st} - K_x^{st} a_2)}{2} z^2 + K_4 Z + L_4 \right], \\
y_{KL}^{dyn}(z, t) &= (\bar{K}_2 \cos \omega_0 z + \bar{L}_2 \sin \omega_0 z + \bar{M}_2 \operatorname{ch} \omega_0 z + \bar{N}_2 \operatorname{sh} \omega_0 z) \sin \omega t, \\
x_{KL}^{dyn}(z, t) &= (\bar{K}_4 \cos \omega_0 z + \bar{L}_4 \sin \omega_0 z + \bar{M}_4 \operatorname{ch} \omega_0 z + \bar{N}_4 \operatorname{sh} \omega_0 z) \cos \omega t.
\end{aligned} \tag{5}$$

full deformation of the upper shaft for the relevant axes;

$$\begin{aligned}
y_{KL\Sigma}(z, t) &= y_{KL}^{st}(z) + y_{KL}^{din}(z, t) \\
x_{KL\Sigma}(z, t) &= x_{KL}^{st}(z) + x_{KL}^{din}(z, t)
\end{aligned} \tag{6}$$

full deflection of the upper shaft for part  $KL$ ;

$$f_{KL}(z, t) = \sqrt{(y_{KL\Sigma}(z, t))^2 + (x_{KL\Sigma}(z, t))^2}. \tag{7}$$

This expression is transformed into the following form to obtain the dependence for calculating the full deflection of the upper shaft.

$$f_{KL}(z, t) = \sqrt{\operatorname{var} KL + 2(\bar{K}_2 \cos \omega_0 z + \bar{L}_2 \sin \omega_0 z + \bar{M}_2 \operatorname{ch} \omega_0 z + \bar{N}_2 \operatorname{sh} \omega_0 z) \cdot [y_{KL}^{st}(z) \sin \omega t + x_{KL}^{st}(z) \cos \omega t]}, \tag{8}$$

where

$$\begin{aligned}
\operatorname{var} KL &= (y_{KL}^{st}(z))^2 + (x_{KL}^{st}(z))^2 \\
&\quad + (\bar{K}_2 \cos \omega_0 z + \bar{L}_2 \sin \omega_0 z + \bar{M}_2 \operatorname{ch} \omega_0 z + \bar{N}_2 \operatorname{sh} \omega_0 z).
\end{aligned}$$

The following symbols are used in the above expressions:

Constants of integration  $K_i$ ,  $L_i$ , ( $i = 1, 2$ ) can be calculated from next expressions:

$$\begin{aligned}
K_1 &= -\frac{[3a + 2(a_2 + b_2) + b_2^2](G_4^e + 2F_4^e + K_y^{st})}{6} \\
&\quad - \frac{(2a_2 + b_2)(M_{G_4x}^e - K_y^{st}a_2) + K_y^{st}a_2^2}{2}, \\
L_1 &= \frac{a_2[3a_2(a_2 + b_2) + b_2^2](G_4^e + 2F_4^e + K_y^{st})}{6} \\
&\quad + \frac{a_2(a_2 + b_2)(M_{G_4x}^e - K_y^{st}a_2)}{2} - \frac{a_2^3(G_4^e + 2F_4^e)}{6}, \\
K_2 &= -\frac{[3a + 2(a_2 + b_2) + b_2^2](G_4^e + 2F_4^e + K_y^{st})}{6} \\
&\quad - \frac{(2a_2 + b_2)(M_{G_4x}^e - K_y^{st}a_2)}{2}, \\
L_1 &= \frac{a_2(a_2 + b_2)(2a_2 + b_2)(G_4^e + 2F_4^e + K_y^{st})}{6} \\
&\quad + \frac{a_2(a_2 + b_2)(M_{G_4x}^e - K_y^{st}a_2)}{2}.
\end{aligned} \tag{9}$$

Constants of integration  $K_j$ ,  $L_j$ , ( $j = 3, 4$ ) can be calculated from expressions (10):

$$\begin{aligned}
K_3 &= \frac{-3(2a_2 + b_2)M_{R_4y}^{st} - b_2^2K_x^{st}}{6}, \\
L_3 &= \frac{3a_2(a_2 + b_2)M_{R_4y}^{st} + a_2b_2^2K_x^{st}}{6}, \\
K_4 &= \frac{(3a_2^2 - b_2^2)K_x^{st} - 3(2a_2 + b_2)M_{R_4y}^{st}}{6}, \\
L_4 &= \frac{a_2(b_2^2 + a_2^2)K_x^{st} + 3a_2(a_2 + b_2)M_{R_4y}^{st}}{6}.
\end{aligned} \tag{10}$$

$G_4^e$ ,  $F_4^e$ ,  $M_{G_4x}^e$  and  $M_{R_4y}^{st}$  are static forces and moments. They can be calculated from the dependences (11) written below.

$$\begin{aligned}
M_{G_4x}^e &= m_4ge \sin \alpha, \quad M_{R_4y}^{st} = \frac{1}{2} \left( \frac{mK_{\Delta(\lambda)}bHu}{V} + R_\Sigma \right) r_4 \cos \alpha, \\
G_4^e &= m_4g, \quad F_4^e = 1.5N_c/V,
\end{aligned} \tag{11}$$

where  $m+4$  and  $r_4$  are the mass and the radius of the leading wheel 4 and  $g$  is the acceleration of gravity.  $R_\Sigma$  is the total resistance force.  $H$  is the thickness

of the workpiece or sawing height,  $u$  is the feeding speed,  $b$  is the width of the cutter.  $m$  is a coefficient ( $0 \leq m \leq 1$ ).  $N$  is the cutting power and  $V$  is the cutting speed.  $K_{\Delta(\lambda)}$  is the specific work of the cutting [11].

$$K_{\Delta} = k + \frac{a_{\rho}p}{u_{z\Delta}} + \frac{\alpha_{\Delta}H}{b}, \quad K_{\lambda} = k + \frac{a_{\rho}ps}{bu_{z\lambda}} + \frac{\alpha_{\lambda}H}{b}. \quad (12)$$

The symbol  $\alpha_{\Delta(\lambda)}$  is the friction intensity of the shavings on the sawing walls, as  $\alpha_{\Delta}$  is used for swage-set teeth and  $\alpha_{\lambda}$  is used for part-set teeth ( $\alpha_{\lambda} = 1.25\alpha_{\Delta}$ ).  $a_{\rho}$  is a coefficient of the blunt teeth, ( $1.4 \leq a_{\rho} \leq 2.8$ ).  $s$  is the thickness of the band saw blades.  $k$  is the fictitious pressure on the front side of the teeth and  $p$  is the fictitious specific force on the back side of the teeth. The feeding of one tooth is marked with  $u_{z\Delta}$ ,  $u_{z\lambda}$ .

$K_x^{st}$  and  $K_y^{st}$  are static reactions. They are caused by static forces and moments and can be calculated from expressions (13).

$$K_x^{st} = -\frac{r_4}{2b_2} \left( \frac{mK_{\Delta(\lambda)}bHu}{V} + R_{\Sigma} \right) \cos \alpha, \quad (13)$$

$$K_y^{st} = -\frac{m_4g}{b_2} (a_2 + b_2 + e \sin \alpha) - \frac{3N_c(a_2 + b_2)}{b_2V}.$$

$\bar{K}_i$ ,  $\bar{L}_i$ ,  $\bar{M}_i$ ,  $\bar{N}_i$ , ( $i = 1, 2$ ) are constants of integration. They are determined by the boundary conditions for each part of upper shaft and can be calculated from expressions below.

$$\bar{K}_i = \frac{\Delta \bar{K}_i}{\Delta_M}, \quad \bar{L}_i = \frac{\Delta \bar{L}_i}{\Delta_M}, \quad \bar{M}_i = \frac{\Delta \bar{M}_i}{\Delta_M}, \quad \bar{N}_i = \frac{\Delta \bar{N}_i}{\Delta_M}, \quad (i = 1, 2), \quad (14)$$

where  $\Delta_M = \det M$  is the determinant of the matrix  $\mathbf{M}$ . This determinant is non-zero, because otherwise the mechanical system will operate in resonance mode and it has the following form:

$$\mathbf{M} = \quad (15)$$

$$\begin{bmatrix} 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \cos \omega_0 a_2 & \sin \omega_0 a_2 & \text{ch } \omega_0 a_2 & \text{sh } \omega_0 a_2 & 0 & 0 & 0 & 0 \\ -\sin \omega_0 a_2 & \cos \omega_0 a_2 & \text{sh } \omega_0 a_2 & \text{ch } \omega_0 a_2 & \sin \omega_0 a_2 & -\cos \omega_0 a_2 & -\text{sh } \omega_0 a_2 & -\text{ch } \omega_0 a_2 \\ -\sin \omega_0 a_2 & \cos \omega_0 a_2 & -\text{sh } \omega_0 a_2 & -\text{ch } \omega_0 a_2 & \sin \omega_0 a_2 & -\cos \omega_0 a_2 & \text{sh } \omega_0 a_2 & \text{ch } \omega_0 a_2 \\ 0 & 0 & 0 & 0 & \cos \omega_0 a_2 & \sin \omega_0 a_2 & \text{ch } \omega_0 a_2 & \text{sh } \omega_0 a_2 \\ 0 & 0 & 0 & 0 & \cos \omega_0(a_2 + b_2) & \sin \omega_0(a_2 + b_2) & \text{ch } \omega_0(a_2 + b_2) & \text{sh } \omega_0(a_2 + b_2) \\ 0 & 0 & 0 & 0 & -\sin \omega_0(a_2 + b_2) & \cos \omega_0(a_2 + b_2) & -\text{sh } \omega_0(a_2 + b_2) & -\text{ch } \omega_0(a_2 + b_2) \end{bmatrix}$$

The other determinants in expressions (14) are determinants of the matrices formed by the matrix  $\mathbf{M}$ , in which the corresponding columns are replaced by the column-vector  $\mathbf{m}$  as follows:

$\Delta_{\bar{K}_i}$  ( $i = 1, 2$ ) – the first or fifth column of the matrix  $\mathbf{M}$ ,  
 $\Delta_{\bar{L}_i}$  ( $i = 1, 2$ ) – the second or sixth column of the matrix  $\mathbf{M}$ ,  
 $\Delta_{\bar{M}_i}$  ( $i = 1, 2$ ) – the third or seventh column of the matrix  $\mathbf{M}$ ,  
 $\Delta_{\bar{N}_i}$  ( $i = 1, 2$ ) – the fourth or eighth column of the matrix  $\mathbf{M}$ .  
 The column-vector  $\mathbf{m}$  has the following form:

$$\mathbf{m} = \left[ \frac{m_4 \omega^2 e \cos \alpha}{E J \omega_0^3} \quad \frac{M_{bm}}{E J \omega_0^2} \quad 0 \quad 0 \quad \frac{K_{bm}}{E J \omega_0^3} \quad 0 \quad 0 \quad \frac{L_{bm}}{E J \omega_0^3} \right]^T, \quad (16)$$

where  $K_{bm}$ ,  $L_{bm}$  and  $M_{bm}$  are calculated from the following dependencies:

$$\begin{aligned}
 K_{bm} &= -\frac{1}{2b_2} \left( \frac{mK_{\Delta(\lambda)}bHu}{V} + R_{\Sigma} \right) e \cos \alpha \\
 &\quad - \frac{\omega^2}{b_2} \left[ (a_2 + b_2 + e \sin \alpha) m_4 e \cos \alpha - \frac{1}{2} (J_{\xi} - J_{\zeta}) \sin 2\alpha \right], \\
 L_{bm} &= \frac{1}{2b_2} \left( \frac{mK_{\Delta(\lambda)}bHu}{V} + R_{\Sigma} \right) e \cos \alpha \\
 &\quad + \frac{\omega^2}{b_2} \left[ (a_2 + e \sin \alpha) m_4 e \cos \alpha - \frac{1}{2} (J_{\xi} - J_{\zeta}) \sin 2\alpha \right] \\
 M_{bm} &= \frac{1}{2} \left[ \left( \frac{mK_{\Delta(\lambda)}bHu}{V} + R_{\Sigma} \right) e \cos \alpha - \omega^2 (J_{\xi} - J_{\zeta} - m_4 e^2) \sin 2\alpha \right].
 \end{aligned} \quad (17)$$

$J_{\xi}$  and  $J_{\zeta}$  are the mass moments of inertia of the leading wheel.  $\omega_0$  is parameter, ( $\omega_0^4 = A_{ms} \rho \omega^2 / EJ$ ), [12, 13]. In this expression  $E$  is the modulus of elasticity,  $J = J_x = J_y$  is the axial moment of inertia,  $A_{ms}$  is the cross-sectional area of the shaft,  $\rho$  is the density of the material.

$\bar{K}_j$ ,  $\bar{L}_j$ ,  $\bar{M}_j$ ,  $\bar{N}_j$ , ( $j = 3, 4$ ) are constants of integration. In this case, the equality between the constants, which is written in expressions (18), must be taken into account.

$$\begin{aligned}
 \bar{K}_3 &= \bar{K}_1, & \bar{K}_4 &= \bar{K}_2, & \bar{L}_3 &= \bar{L}_1, & \bar{L}_4 &= \bar{L}_2, \\
 \bar{M}_3 &= \bar{M}_1, & \bar{M}_4 &= \bar{M}_2, & \bar{N}_3 &= \bar{N}_1, & \bar{N}_4 &= \bar{N}_2,
 \end{aligned} \quad (18)$$

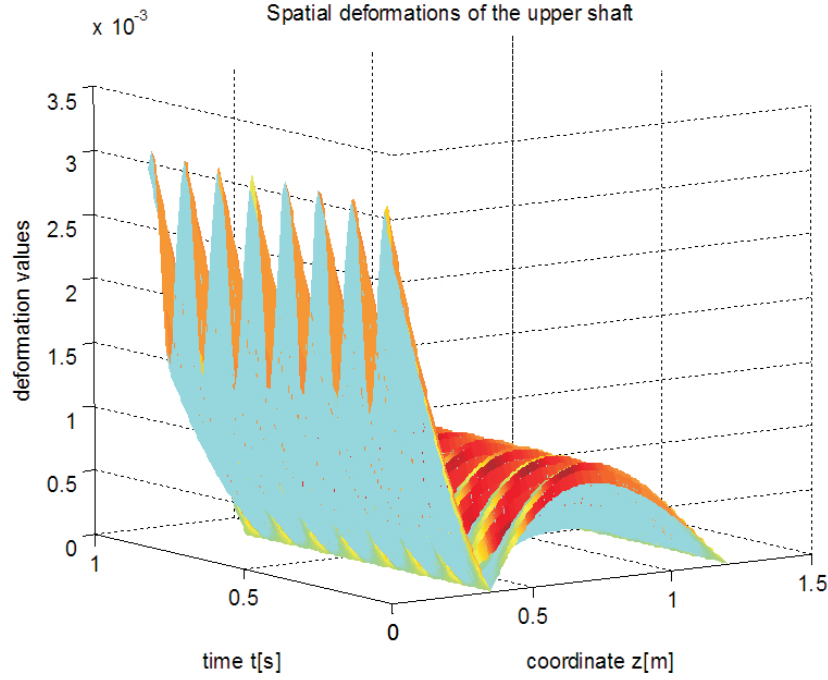
where  $\bar{K}_i$ ,  $\bar{L}_i$ ,  $\bar{M}_i$ ,  $\bar{N}_i$  ( $i = 1, 2$ ) are constants of integration in vertical plane  $O_{yz}$ .

#### 2.4. Deformation checks – numerical solution

The theoretical expressions obtained above are used for numerical solution of the assigned problems. The spatial deformations of the whole shaft can

be obtained by concatenating the expressions describing the full deflections of the shaft for parts  $O_4K$  and  $KL$ . The following initial data are used [9, 11, 14–18]:

$\omega = 56 \text{ [s}^{-1}\text{]}$ ,  $\omega_0 = 0.6971 \text{ [1/ m]}$ ,  $e = 0.001 \text{ [m]}$ ,  $\alpha = 0.01744 \text{ [rad]}$ ,  $a_2 = 0.4 \text{ [m]}$ ,  $b_2 = 0.8 \text{ [m]}$ ,  $g = 9.81 \text{ [m/s}^2\text{]}$ ,  $m_4 = 810 \text{ [kg]}$ ,  $J_\xi = 171 \text{ [kg m}^2\text{]}$ ,  $J_\zeta = 336 \text{ [kg m}^2\text{]}$ ,  $J_x = J_y = J = 321.8 e-008 \text{ [m}^4\text{]}$ ,  $A_{ms} = 63.585 e-004 \text{ [m}^2\text{]}$ ,  $N_c = 43.4 \text{ [kW]}$ ,  $V = 45 \text{ [m/s]}$ ,  $u = 0.5 \text{ [m/s]}$ ,  $r_4 = 0.8 \text{ [m]}$ ,  $s = 1.47 \text{ [mm]}$ ,  $b = 2.5 \text{ [mm]}$ ,  $p = 9220 \text{ [N/m]}$ ,  $k = 35 \times 10^6 \text{ [Pa]}$ ,  $\alpha_\lambda = 245 \times 10^3 \text{ [Pa]}$ ,  $\alpha_\Delta = 196 \times 10^3 \text{ [Pa]}$ ,  $m = 0.5$ ,  $H = 0.42 \text{ [m]}$ ,  $u = 0.5 \text{ [m/s]}$ ,  $u_{z\Delta} = 2.5 \text{ [mm]}$ ,  $K_\Delta = 80 \times 10^6 \text{ [J/m}^3\text{]}$ ,  $R_\Sigma = 2400 \text{ [N]}$ ,  $\rho = 7850 \text{ [kg/m}^3\text{]}$ ,  $F_4^e = 1450 \text{ [N]}$ ,  $R_\Sigma = 2400 \text{ [N]}$ ,  $P_u = 2850 \text{ [N]}$ ,  $G = 7946 \text{ [N]}$ ,  $E = 2.06 e + 011 \text{ [Pa]}$ .



**Fig. 3.** Spatial deformations of the upper shaft

We apply the algorithm for numerical determination of the maximum deflection of the shaft. This algorithm is presented in section 2.3. Obviously, the most endangered cross-section is situated in the beginning of the shaft, where  $z = 0$ . We denote  $f_{O_4K}(0, t) = \max f_{O_4K}$ . That is why, the expression (4) can

be written as follows.

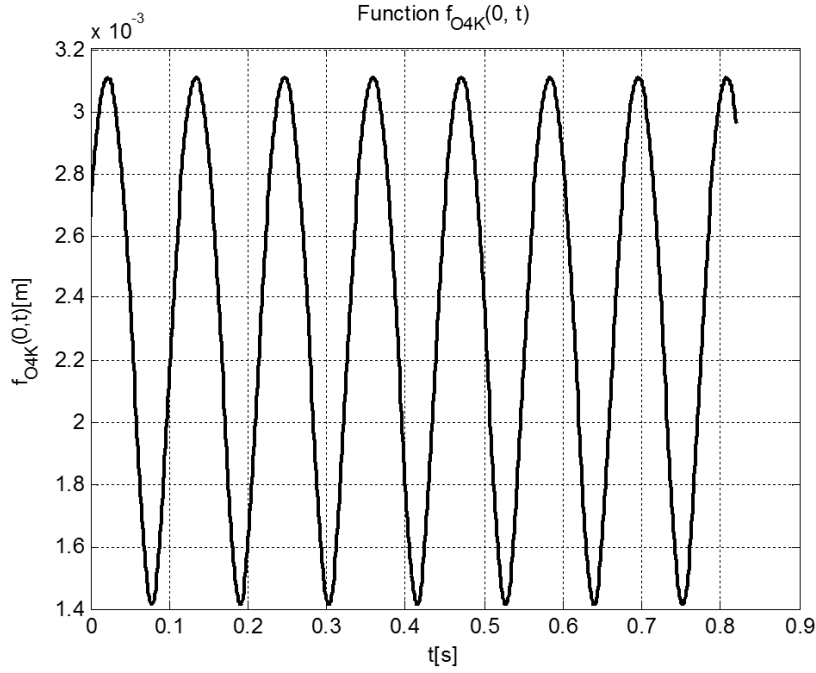
$$f_{O_4K}(0, t) = \max f_{O_4K} \\ = \sqrt{(L_1^2 + L_3^2)/(EJ)^2 + (\bar{K}_1 + \bar{M}_1)^2 + 2(\bar{K}_1 + \bar{M}_1)(L_1 \sin \omega t + L_3 \cos \omega t)/(EJ)}. \quad (19)$$

The maximum value of the upper expression must be determined. This can be performed in different ways, which are presented below.

a) calculation of the maximum values of the function in the area of local maximum;

We can proceed in the following way:

Time domain where the function  $f_{O_4K}(0, t)$  has a local maximum must be determined. For this purpose, the diagram of the function is shown in Fig. 4. From this figure, we can assume that the first local maximum appears in the interval



**Fig. 4.** Diagram of the function  $f_{O_4K}(0, t)$

The function values are calculated in the defined interval. The resulting data are represented as a matrix. The maximum deflection of the shaft is

selected from the matrix below. Precise calculations show the exact value of the maximum deflection  $\max f_{O_4K} = 0.00310849\ 637733$  [m]. This value is used to perform the deformation check of the upper shaft but it is not necessary for practical calculations. For this reason, we can assume  $\max f_{O_4K} = 0.00310849$  [m].

$$f_{O_4K} = \begin{bmatrix} \text{Columns 1 through 5} \\ 0.00212497 & 0.00217236 & 0.00221952 & 0.00226632 & 0.00231265 \\ \text{Columns 6 through 10} \\ 0.00235838 & 0.00240342 & 0.00244767 & 0.00249103 & 0.00253341 \\ \cdot \\ \cdot \\ \text{Columns 31 through 35} \\ 0.00309076 & 0.00309808 & 0.00310348 & 0.00310695 & 0.00310849 \\ \cdot \\ \cdot \\ \text{Columns 91 through 95} \\ 0.00141307 & 0.00141350 & 0.00141818 & 0.00142705 & 0.00143999 \\ \text{Columns 96 through 100} \\ 0.00145687 & 0.00147749 & 0.00150164 & 0.00152908 & 0.00155954 \end{bmatrix}$$

b) second way: use of optimization procedures for finding the maximum of the function.

We use the optimization procedure *fminbnd* [19, 20]. This procedure calculates minimum of a function that depends on one argument. That is why the expression (19) must be examined with a negative sign. The obtained value for  $\max f_{O_4K}$  must be chosen with a positive sign. It is necessary to determine the boundaries in which the time changes, i.e.  $t_1 = 0.1$  [s],  $t_2 = 0.2$  [s].

```
function F=def1(x)
[x,fval,exitflag,output]=fminbnd(@def2,0.1,0.2,optimset('TolX',1.e-12))
t = 0.13429722095161 [s],  $\max f_{O_4K} = 0.00310858179145$  [m]
exitflag =1
output =
algorithm: 'golden section search, parabolic interpolation'
```

The argument “output” contains records of the results from the optimization. The most important record shows the algorithm for finding the maximum of the function.

It is obvious that the calculated result for  $\max f_{O_4K}$  is the same as this one in the first way to find the maximum of the function. This deflection is the biggest deflection. It is denoted by  $F_{\max}$  and can be used to perform a deformation check of the upper shaft. In this case, the following expressions are used [4, 5, 6]:

$$f_{\max} \leq [f], \quad f_{\max}/l_b \leq [f/l_b], \quad (20)$$

where  $[f]$  is the admissible deflection and  $[f/l_b]$  is the admissible relative deflection. The values of this deflection are taken from technical literature. We can choose the most appropriate value for each particular case, i.e.,  $[f/l_b] = a_b$ . The admissible deflection can be calculated in the following way:  $[f] = a_b l_b$ . The distance  $l_b$  accepts different values for the different parts: for part  $O_4K$ ,  $l_b = a_2$ , for part  $KL$ ,  $l_b = b_2$ . The deformation conditions are satisfied if the maximum deflection  $f_{\max}$  or the maximum relative deflection  $f_{\max}/l_b$  are smaller or equal to the respective admissible deflections. The final expressions are shown below.

$$f_{\max} \leq a_b l_b, \quad f_{\max}/l_b \leq a_b. \quad (21)$$

### 3. CONCLUSION

In this study, an algorithm for performing deformation checks in upper shaft of big band saw machines is proposed. The article consists of two main parts. The first part is entitled “Deformation checks-analytical solution”. In this part, theoretical expressions by which shaft deflections can be calculated in all parts (parts  $O_4K$  and  $KL$ ) are obtained. For this purpose, analytical expressions, which describe the deformations of the upper shaft of the big band saw machines in two mutually perpendicular planes are used.

The second part is entitled “Deformation checks-numerical solution”. In this part, two main ways of determining the most endangered section are considered. Each a way possesses its advantages and disadvantages. These ways require relevant calculations to be performed, as well as diagrams and surfaces to be drawn and optimization procedures to be included. For this purpose, the computer programs need to be created. These programs allow solving various optimization problems taking into account various parameters. The purpose of these solutions is the spatial deformations of the transmissions to have minimum values.

In conclusion we can say that this study can be applied for deformation checks in the transmissions of other classes of woodworking machines. The

developed theory can be used in the design process of new woodworking machines. These machines must be characterized by high reliability and safety in operating mode.

## REFERENCES

- [1] B. MARINOV, Investigation OF THE Spatial Deformations and Transverse Vibrations in Main Links of Big Band Saw Machines, *Journal of Theoretical and Applied Mechanics* (2024) **54** (1) 73–88, Print ISSN: 0861-6663, Online ISSN: 1314-8710, online: <https://doi.org/10.55787/jtams.24.54.1.073>.
- [2] B. MARINOV, Spatial deformations and transverse vibrations in the cutting mechanisms of big band saw machines: Investigation and analysis, *Global Journal of Engineering and Technology Advances* (2022) **13** (3) 96–109, ISSN: 2582-5003, online: <https://doi.org/10.30574/gjeta.2022.13.3.0211>.
- [3] B. MARINOV, *Energy Losses in Big Woodworking Machines: Analysis and Optimization*, Cambridge Scholars Publishing (2023), ISBN: 1-5275-1533-8, ISBN13: 978-1-5275-1533-8.
- [4] I. KISYOV, *Strength of Materials*, State Publishing House “Technique” (1978).
- [5] V. MILKOV, *Strength of Materials, Part I* Offset-Printed Base at TU Varna (2008).
- [6] V. MILKOV, *Strength of Materials, Part II* Offset-Printed Base at TU Varna (2010).
- [7] E. HEARN, *Mechanics of Materials 1–2: An Introduction to the Mechanics of Elastic and Plastic Deformation of Solids and Structural Materials*, 3rd edition, Printed and Bound in Great Britain by Scotprint, Musselburgh (1997).
- [8] F. BEER, E. RUSSELL, J. T. DEWOLF, AND D. F. MAZUREK, *Mechanics of Materials*, 5th edition, Published by McGraw-Hill, New York, NY 10020 (2009).
- [9] P. OBRESHKOV, *Woodworking Machines*, Publishing House “BM” (1995).
- [10] B. MARINOV, Deformation Checks in the Transmissions of Certain Classes of Woodworking Machines, *International Journal of Research in Mechanical Engineering* (2015) **3** (5), 28–41.
- [11] ZH. GOCHEV, *Handbook for Exercise of Wood Cutting and Woodworking Tools*, (2005), Publishing House in LTU.
- [12] B. CHESHANKOV, *Theory of Vibrations*, Publishing House of Technical University (1992).
- [13] A. PISAREV, *Mechanical Vibrations*, State Publishing House Technique (1985).
- [14] Š. BARČÍK, Experimental cutting on the table band-sawing mashine, *European Journal of Wood and Wood Products* (2003) **61** (4) 313–320, online: <https://doi.org/10.1007/s00107-002-0342-9>.
- [15] R. BRUCE, *Understanding Wood: A Craftsman’s Guide to Wood Technology* Newtown, Connecticut, Taunton Press (1995).
- [16] M. YAN, *Journal of Forestry Machinery & Woodworking Equipment* (1998) **8** 4–8.

- [17] P. GENDRAUD, J. C. ROUX, AND J. M. BERGHAU, *Journal of Materials Processing Technology* (2003) **135** 109–116.
- [18] R. OKAI, S. KIMURA, AND H. YOKOCHI, *Journal of the Japan Wood Research Society* (1996) **42** 333–342.
- [19] Y. TONCHEV, *Matlab*, Part 1, State Publishing House “Technique” (2005).
- [20] Y. TONCHEV, *Matlab*, Part 3, State Publishing House “Technique” (2009).

*Received November 15, 2024*