THE APPLICABILITY OF QUANTUM GYROSCOPES FOR NAVIGATION

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Abstract. The report examines the possibilities of using atomic gyros for an aircraft. Gauge formulas are given for a fiber optic gyroscope and an atomic gyroscope. Simplified calculations were made for their measurement and sensitivity ranges. Based on these, comparative conclusions are drawn.

Keywords: cold atom sensors, atomic gyroscope, atomic interferometer, satellite free navigation.

1. INTRODUCTION

The theoretical possibility for aviation gravimetry described in [2] raises a number of questions. Are there sufficiently accurate sensors to measure linear accelerations and angular velocities to be used for air navigation? How to use mathematical models of inertial sensor errors in Gravitation Measurement System (GMS) algorithms [1]? How to use second derivatives of gravitational potential? Are the Eötvös effect and the Faye anomaly in the open air sufficiently accurately taken into account during the flight [1], [3]? The answers to these questions are still being sought. New technical devices come into being, first as laboratory models and then as industrial products. Research begins with the development of new methods for measuring linear accelerations and angular velocities and continues with the improvement of technical instruments, the study of their errors in measuring the relative force of gravity, and the speed of the Earth's rotation around the clock. New methods and algorithms for complex information processing by multiple sensors are being created.

The traditional notion of a gyroscope, as an instrument sensitive to angular velocity, is always associated with the idea of having a symmetrical fast-rotating rotor suspended so that its center of mass coincides with the geometric center of the suspension. The presence of a massive, rapidly rotating, or vibrating body is not necessary to register the angular velocity of rotation. Atomic nuclei or electrons having spin and orbital moments can be used. These devices are called quantum gyroscopes.

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The Sagnac interferometer is the basis of all Fibre Optic Gyroscopes (IFOG), which are becoming more and more widely used for air and sea navigation. The most famous of the optical gyroscopes is the active resonant Ring Laser gyroscope (RLG). Its successful production dominated air navigation systems after 1980, after which it was gradually replaced by IFOG.

The atomic interferometer gyroscope (AIG) and the atomic spin gyroscope (ASG) measure both angular velocity and linear acceleration and can also measure gravitational and magnetic fields. They are a product of cold atom technology and are still in the laboratory mock-up phase or are too bulky and heavy for air navigation applications. Basic research in this field is gaining momentum after four consecutive Nobel Prizes in Physics.

In 1997, the award went to Claude Cohen-Tannoudji and William D. Phillips "for developing methods for cooling and capturing atoms with laser light". The award went to Eric A. Cornell, Wolfgang Keterle, and Carl E. Wiemann "for achievements in Bose-Einstein condensation in dilute gases of alkaline atoms and for earlier basic research on the properties of condensates" in 2001. It was shared by Roy J. Glauber "for his contribution to the theory of quantum optical coherence" John L. Hall and Theodor W. Hänsch "for their contribution to the development of precision laser spectroscopy, including the optical frequency comb technique" in 2005. The 2017 Nobel Prize in Physics was awarded to Rainer Weiss, Barry C. Barish, and Kip S. Thorne "for their decisive contribution to the LIGO detector and the observation of gravitational waves".

2. PRINCIPLE OF QUANTUM INTERFEROMETERS

Georges Sagnac reported in 1913-1914 that he had seen the movement of interference lines derived from two beams propagating oppositely in a circular optical path. Circular interferometer, the two light waves circulate in opposite directions on a circular contour with radius R, starting from the source S is shown in Fig. 1. Let the source rotate at speed Ω and c is the speed of light propagation. Light passing in the opposite direction of rotation will return to S earlier than that, moving in the same direction. A wave propagating in the direction of rotation travels the path L^+ in time t^+ and the opposite wave travels L^- in time t^-. The roads passed are:

$$L^+ = ct^+ = 2\pi R + R\Omega t^+,$$

$$L^- = ct^- = 2\pi R - R\Omega t^-.$$

The differences in the propagation times of the two rays are:

$$t^+ - t^- = 2\pi R\left(\frac{1}{c-R\Omega} - \frac{1}{c+R\Omega}\right) = \frac{4\pi R^2\Omega}{c^2-(R\Omega)^2}.$$

If the light waves pass N times along the optical path and considering that the speed of light is much higher than the linear speed of movement of the source $c >> R\Omega$, the difference in propagation times is:

$$\Delta t = N(t^{+} - t^{-}) = 4\pi N R^2 \Omega / c^2 = (4NA/c^2)\Omega.$$

The difference in distance, expressed by the difference in propagation times, is:

$$\Delta L = c\Delta t = (4NA/c)\Omega = (Ld/c)\Omega \tag{1}$$

where: A is the area closed by the optical path; d is path diameter and L is the total length traversed.

For a wave of frequency f (wavelength $\lambda = c/f$), the period $T = 1/f$ is the time taken for a 2π phase change so that over time Δt phase change Φ is:

$$\Phi = 2\pi \Delta t / T = 2\pi f \Delta t.$$

After substituting for Δt:

$$\Phi = (8\pi NA/\lambda c)\Omega \Leftrightarrow \Omega = \frac{\Phi \lambda c}{8\pi NA} \tag{2}$$

The fringe shift, x, is then $\Phi = 2\pi$, and:

$$x = (4NA\lambda c)\Omega = (4Ld/\lambda c)\Omega = K_0 \Omega \tag{3}$$

where $K_0 = 4Ld/\lambda c$ is the open-loop scale factor.

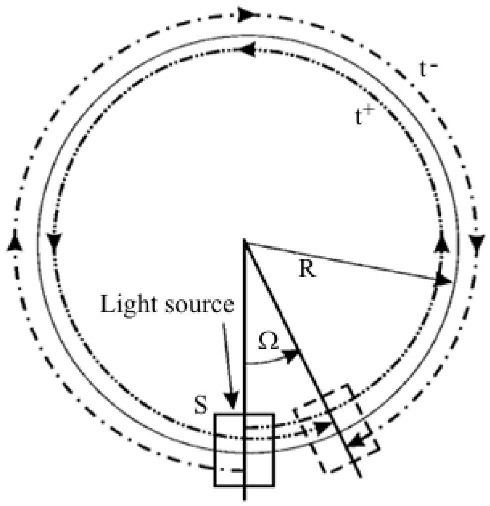

Fig. 1. Difference in the times for passing the left and right light beam when rotating the optical path

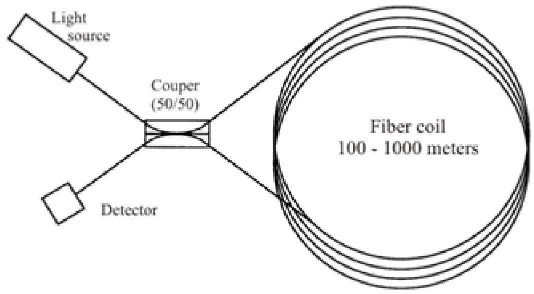

Fig. 2. Basic scheme of laser fiber optic gyroscope

2.1 Fiber Optic Gyroscope (FOG)

Figure 2 shows the basic diagram of FOG, which is a fiber-optic analog of the Sagnac interferometer. Light from a coherent source enters the 50% splitter, splitting into two waves in two opposite directions, passing through the coil of fiber. When the light returns to the splitter, the waves merge and separate, half returning to the source and half to the photodetector. The intensity of the light in the sensor is an indicator of the interference between the two light beams and, thus, of the speed of rotation of the coil in its plane. The intensity may vary for other reasons, such as temperature variations in the power of the light source or in the splitter, and it is necessary to make appropriate compensations for these effects.

To calculate the measuring range, it is assumed that the winding has an area of $100cm^2$ with $N=1000$ turns and operates at a wavelength of $1{,}3\mu m$. Then, according to equation (2), the zero point ($\Phi = \pi/2$) will have an angular velocity accuracy, which, without feedback, will be approximately:

$$\Omega = \lambda c/16NA \approx 150 deg/s \approx 0{,}042º/h.$$

This is not enough. Most vehicles can be subjected to short-term angular velocities up to *250deg/s* and others up to *500deg/s*. One solution is to reduce the winding area A and/or the number of turns N, but for low resolution, it is necessary to have a large value of AN (Table 1). Thus, IFOG without feedback can only be used in low-precision inertial systems or in the Attitude Heading Reference System (AHRS), where the measured angular velocities are small.

A high-quality aeroplane gyroscope must have a resolution and zero-point stability of less than *0,01°/h*. Aircraft gyroscopes are generally subject to very high requirements [5]: zero resolution and drift of *0,01°/h*, dynamic range of 6 orders of magnitude, and high stability (10^{-5}) of the scale conversion factor K_0 from equation (3).

Equation (1) gives the relationship between the difference in light path, representing the Sagnac phase, and the speed of rotation. If we assume that $d = 10cm$ and

$L = 1km$, then at two values of the angular velocity Ω, the absolute and relative changes between the light paths are given in Table 1:

Table 1. Sensitivity of FOG

Angular velocity Ω	ΔL	$\Delta L/L$
1 [deg/h]	10^{-10}	10^{-13}
1 [deg/s]	3×10^{-7}	3×10^{-10}

To imagine the size of these ΔL values, we can compare them with the diameter of the hydrogen atom, which is approximately 1,7Å or $1{,}7\times10^{-10}m$. To measure with such accuracy, the left and right light paths must have an almost perfect uniform shape, they must be absolutely reciprocal.

2.2 Atomic Interferometric Gyroscope (AIG)

Atom interferometry uses the coherent splitting and recombination of atoms to make precision measurements of environmental parameters such as gravity, acceleration, or magnetic field.

The principle of operation of AIG for measuring the absolute angular velocity of rotation is shown in Fig. 3. The atomic interferometer uses the wave nature of atoms. Like optical interferometers, atomic interferometers measure the phase shift of two

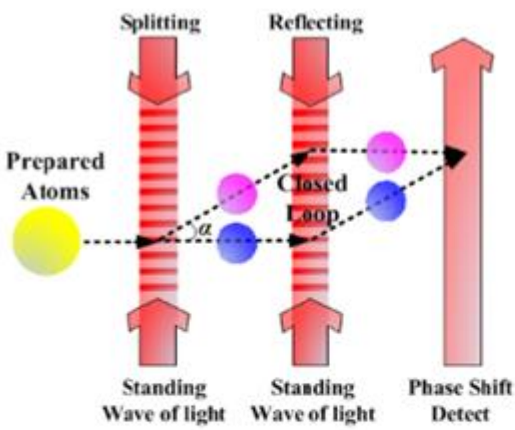

Fig. 3. AIG sensitivity to relative weight and absolute angular velocity

material waves of atoms passing through different paths (Fig. 4). The waves of matter are of cold atoms. The atom, as well as the photon, exhibit properties of both wave and particle. Thanks to the Sagnac effect, the atomic interferometer can work like a gyroscope, just like the optical gyroscope. Cold atoms separate at the starting point and gather at the endpoint, forming a closed loop. At the endpoint, there will be a phase shift in the presence of an absolute angular velocity. Ω. The basic principle of AIG is similar to that of the optical gyroscope, but it has a different design because the "separator", "mirror," and "detector" are different. As can be seen in Fig. 4, a portion of the atoms are prepared so that they are in a suitable state and directed to the interferometer. The flow of atoms is manipulated by standing light waves. This light is emitted by lasers and acts as a "separator" or "mirror" for atoms in a coherent superposition of the momentary state. The movement of atoms through these light grids forms a closed circle. The third laser beam is used to separate the phase shift of the flux of atoms and thus determine the value of the speed of rotation.

The rotation of the AIG housing causes a phase shift $\Phi = \frac{4\pi m}{h}\vec{\Omega} \cdot \vec{A}$, where m is the mass of the atom, h is the Planck constant, $\vec{\Omega}$ is the rotation rate vector, and $\vec{A}$ is the closed loop area vector. For comparison with the optical gyroscope, equation (2) accepts the form: $\Phi_{FOG} = \frac{4\pi}{\lambda c}\vec{\Omega} \cdot \vec{A}$, where λ is the wavelength of light and c is the speed of light.

The comparison between the sensitivity of AIG and FOG is: $\frac{\delta\Phi}{\delta\Phi_{FOG}} = \frac{m\lambda c}{h} > 10^{10}$. Therefore, atomic gyroscopes are at least 10^{10} times more sensitive than optical ones at the same closed-loop surface because the mass of the atom is much larger than the relative mass of the photon $(h/\lambda c)$. The theoretical sensitivity of AIG is extremely high but reached by 2006 in the range of 6,8.10-5°/h. In the comparison between the two types of gyroscopes, it should be noted that the closed measuring surface has an area of about 20 mm^2, which is several times less than that of the optical gyroscopes used. In this case, the atoms pass only once through the closed loop because it is too difficult to do this many times, unlike the light beam in FOG. Also, the angle α in Fig. 3 is small, and hence the closed area.

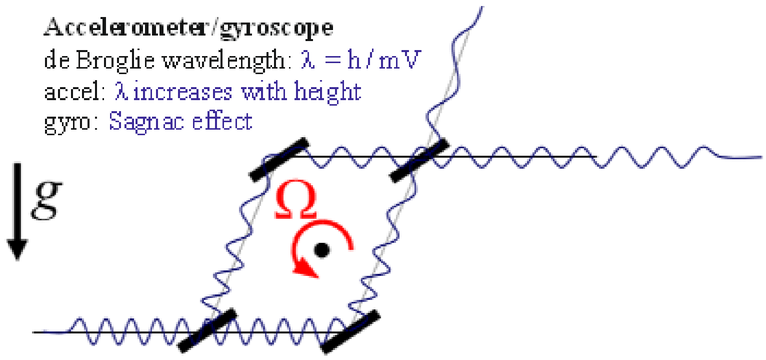

Fig. 4. The AIG sensitivity to relative weight and absolute angular velocity [4]

The limited closed area of AIG is a problem for the construction of gyroscopes with a larger measuring range, but there are other problems with their application in inertial systems [4]. The Inertial Measurement Unit (IMU) for strap-down systems must measure by three mutually perpendicular axes. The AIG of Fig. 4 is suitable for gravimetric measurements of $\vec{g}$. When placing these gyroscopes in inertial blocks, the formula for the output signal is:

$$\Phi = \frac{4\pi m}{h}\vec{\Omega}\cdot\vec{A} + \vec{k}_{eff}\cdot\vec{g}T^2 - \vec{k}_{eff}\cdot\left(\vec{g}\times\vec{\Omega}\right)T^3 \tag{4}$$

where $\vec{k}_{eff}$ is the effective light propagation vector, $\vec{g}$ is the gravitational acceleration, and T is the atom propagation time (Fig. 3). Most importantly, the typical phase shift caused by acceleration is 5–6 orders of magnitude greater than that caused by rotation [5].

3. DISCUSSION AND CONCLUSION

The development of quantum optics and the theory of the cold atom, marked by a number of Nobel Prizes, opened new horizons for theoretical and experimental physics. Cold atom technology will lead to revolutionary technological solutions in aviation. Fig. 5 shows that only high-quality and expensive micromechanical gyroscopes (MEMS) can measure the Earth's diurnal rotation (15°/h) and, therefore, be used for air navigation along with other radio navigation systems. Optical gyroscopes (FOG and RLG) have reached technological excellence and are used in autonomous navigation systems with a location error of less than 1 nautical mile per hour. AIG is an ultra-high precision gyroscope for strategic grade inertial navigation in the future. It takes up the range below 0.00015°/h for the lower threshold of the measured absolute angular velocity [5].

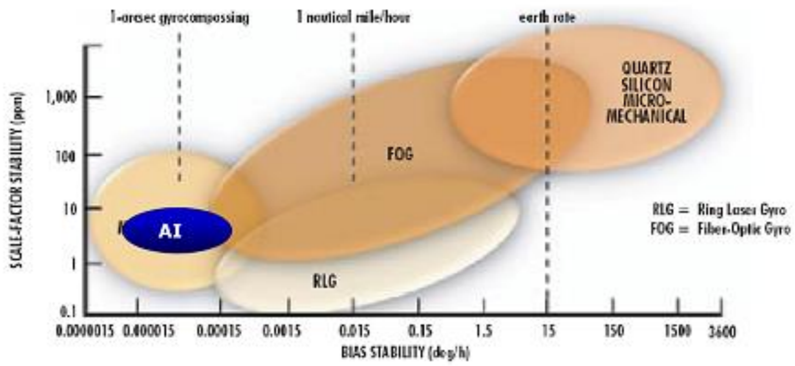

Fig. 5. Comparison of AIG with other gyroscopes for air navigation [5]

The main thing that is expected from atomic interferometers (AI) is to achieve the accuracy of autonomous air navigation only with inertial sensors, without the use of GPS or other radio navigation systems. It is called "satellite-free navigation". This problem could be solved only with highly accurate mathematical models of the Earth's gravitational and magnetic fields. These models can appear only after precise gravimetric and magnetic measurements are performed both on the earth's surface and at a height above it [6]. This is the fundamental direction for the development of devices using cold atom technology.

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REFERENCES

[1] A. MADZAROV, Gyroscopes and Inertial Navigation Systems, D. Metropolia (2000) 422, ISBN 954-713-046-3 (in Bulgarian).

[2] A. MADJAROV, P. PANOVA AND P. GETSOV, Theoretical Research of Possibilities for Gravitation Measurements in Motion Using the Inertial Navigation System (2004) 5133-5138 55th International Astronautical Congress – Vancouver, Canada, Symposium: Microgravity engineering sciences, Manuscript document IAC-04-J.P.08.pdf.

[3] A. MADZHAROV AND S. BILIDEROV, The Influence of the Free Air Anomaly Vver the Stability of the Vertical Channel in Strapdown Inertial Navigation System (SINS) (2006) Proceedings of First International Conference on Cartography & GIS, Borovets, Bulgaria, ISBN 10: 954-724-028-5 and ISBN 13: 978-954-724-028-5.

[4] J. FANG AND J. QIN, Advances in Atomic Gyroscopes: A View from Inertial Navigation Applications (2012) *Sensors*, **12** (5) 6331-6346; online: http://dx.doi.org/10.3390/s120506331, ISSN 1424-8220.

[5] M. TRAVAGNIN, Cold Atom Interferometry for Inertial Navigation Sensors. Technology Assessment: Space and Defence Applications (2020) EUR 30492 EN, JRC122785, Publications Office of the European Union, Luxembourg, ISBN 978-92-76-27076-8, ISSN 1831-9424, online: http://dx.doi.org/10.2760/237221.

[6] A. KOLEV, Software Solutions for 3D Geomodelling, Chief Information Officer – Sofia: IDG Bulgaria (2005) **XV** (7) 91-94, (2019), ISSN 1312-5605.

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