PUNCHING SHEAR RESISTANCE OF CONCRETE FLAT SLABS

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Abstract. Usual cases of support of concrete flat slabs are considered in this article. The limit value of punching force carried by the reinforcement is analyzed. The size of the zone with need of transverse reinforcement for different cases of local support was investigated. The capacity of concrete compression diagonals has been studied in detail. The specific solutions of punching are related to different values of stresses. Practical advice for possible diameters of transverse reinforcement is given.

Keywords: punching, punching shear resistance, shear reinforcement, flat slabs.

1. INTRODUCTION

In the present investigation of punching for flat slabs, it is assumed that a range of effective height of the upper reinforcements above the support of $d = 150 \div 250mm$. This range allows the use of shear reinforcement, if necessary (minimum required flat slab thickness is $200mm$). The shear reinforcement is assumed to be only with vertical branches (double-headed studs, stirrups, continuous cages, etc.).

It is assumed usual concrete classes between $C20/25$ and $C30/37$, and the range of reinforcement ratios of the upper reinforcement $\rho_l = 0{,}6 \div 2\%$.

The presence of openings in the slabs near the supports (columns and walls) is not studied.

A similar problem was considered in [5], but with rather limited parameters and only for inner columns. The results are reported and confirmed here as well, in order to obtain a comprehensive picture of the research phenomenon of punching.

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2. THEORITICAL BACKGROUND

According to Eurocode 2 [1, 2], shear resistance should be checked at the basic control perimeter u_1 (at a distance $2.d$ from support). Punching shear reinforcement is not necessary if:

$$\nu_{Ed} = \frac{\beta . V_{Ed}}{u_1 . d} \leq \nu_{Rd,c} \quad (1)$$

where: β is a coefficient accounting eccentricity of force V_{Ed} compared to the control perimeter u_1.

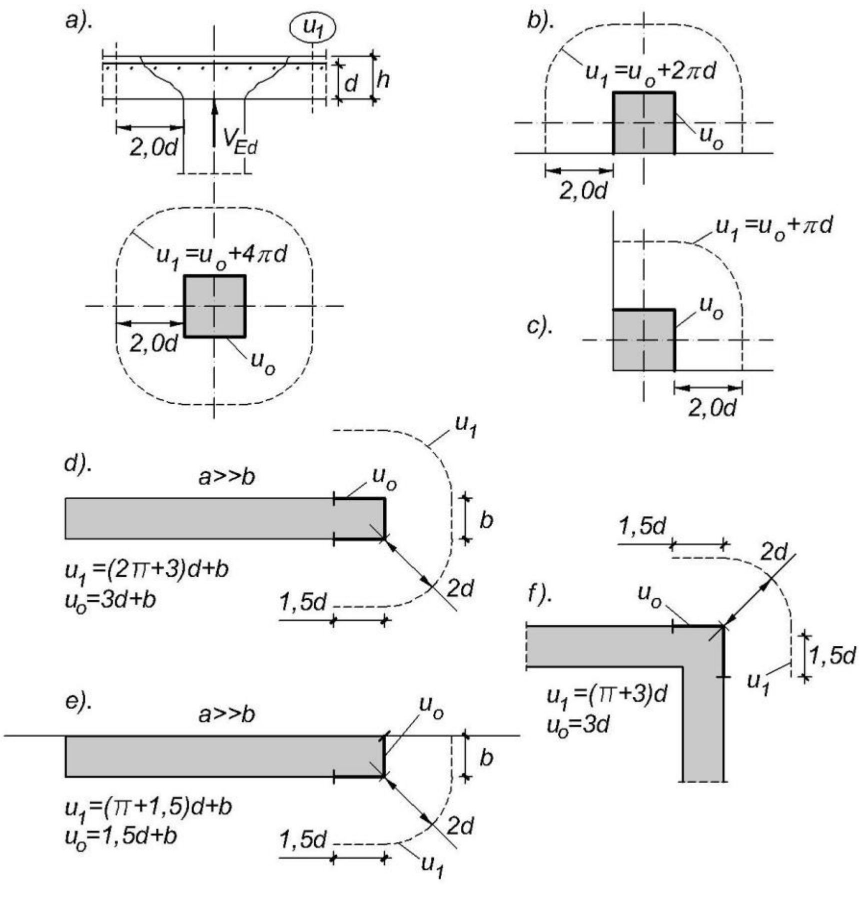

Fig. 1. Base and control perimeter for (a) inner columns, (b) edge columns (c) corner columns and (d) inner walls, (e) edge walls, and (f) wall corners [1, 3, 4, and 5]

If equation (1) is not fulfilled, punching shear resistance of slabs must be ensured by adding shear reinforcement, while relying on a reduced contribution from the concrete:

$$v_{Rd} = 0{,}75.v_{Rd,c} + \frac{1{,}5d}{s_r} A_{sw}.f_{ywd,ef} \frac{1}{u_1.d} \leq 1{,}5.v_{Rd,c} \quad (2)$$

where: $v_{Rd,c} = 0{,}12k(100\rho_l.f_{ck})^{1/3} \geq v_{min}$ is punching shear resistance without shear reinforcement (f_{ck} is in MPa); $v_{min} = 0{,}035.k^{3/2}f_{ck}^{1/2}$ – this value is not used in the investigation, because it is valid only for low value of reinforcement ratio; $k = 1 + \sqrt{200/d}$ – scale factor (d is in mm) – assumed conservative value $k = 2$ in the investigation; $\rho_l = \sqrt{\rho_x.\rho_y} \leq 0{,}02$– reinforcement ratio of longitudinal upper reinforcement; $s_r \leq 0{,}75d$ – the radial spacing of perimeters of shear reinforcement; A_{sw} – the area of one perimeter of shear reinforcement around the support; $f_{ywd,ef} = 250 + 0{,}25d \leq f_{ywd}$– the effective design strength of the punching shear reinforcement (d is in mm); $sin\,\alpha = 1$ – shear reinforcement only with vertical branches.

The expression after the inequality in eq. (2) limits the part of the punching force that can be accepted by shear reinforcement [4, 5] – no more than the reduced contribution of the concrete. This means that if transverse reinforcement is required for the control perimeter, the dependency must also be satisfied:

$$v_{Ed} \leq 1{,}5.v_{Rd,c} \quad (3)$$

Otherwise, shear reinforcement is not allowed, and other measures must be taken to satisfy (1) and (3).

3. ANALYTICAL RESEARCH AND RESULTS

3.1 Determination of the transverse reinforcement zone at maximum punching capacity for various supports

For all perimeters that are further from the supports than the control perimeter u_1, it is assumed that without transverse reinforcement in them, a shear capacity equal to $v_{Rd,c}$ can be achieved.

Then it's possible to determine a perimeter u_{out} (at distance r_{out} and similar in shape to the control perimeter) where shear reinforcement is not already necessary (Fig. 2):

$$u_{out} = \frac{\beta.V_{Ed}}{v_{Rd,c}.d} \quad (4)$$

According to [4, 5], condition (3) can also be represented purely geometrically by the length of this perimeter u_{out}:

$$u_{out} \leq 1{,}5.u_1 \tag{5}$$

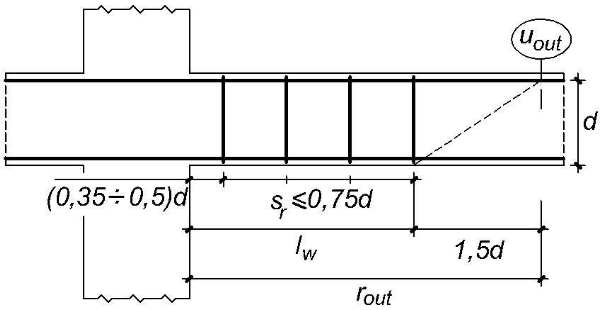

Fig. 2. Shear reinforcement region [3, 4]

The shear reinforcement must extend at least the distance:

$$l_w = r_{out} - 1{,}5.d \tag{6}$$

On the grounds of (5) for different supports is determined allowed distance r_{out}/d, corresponding to perimeters u_{out}, exactly 1,5 times longer than u_1. For these perimeters, according to (6), the lengths of the zones with shear reinforcement l_w are also determined, presented in Table 1 as ratios l_w/d.

Table 1. Shear reinforcement zone l_w/d

	Inner columns	Edge columns	Corner columns
$u_1 =$	$u_0 + 4\pi.d$	$u_0 + 2\pi.d$	$u_0 + \pi.d$
$u_1/d =$	$\frac{u_0}{d} + 4\pi$	$\frac{u_0}{d} + 2\pi$	$\frac{u_0}{d} + \pi$
$u_{out} =$	$u_0 + 2\pi.r_{out}$	$u_0 + \pi.r_{out}$	$u_0 + \frac{\pi.r_{out}}{2}$
$\frac{r_{out}}{d} \leq$	$\frac{0{,}5\frac{u_0}{d} + 6\pi}{2\pi}$	$\frac{0{,}5\frac{u_0}{d} + 3\pi}{\pi}$	$\frac{\frac{u_0}{d} + 3\pi}{\pi}$
$\frac{l_w}{d} \leq$	$\frac{0{,}5\frac{u_0}{d} + 6\pi}{2\pi} - 1{,}5$	$\frac{0{,}5\frac{u_0}{d} + 3\pi}{\pi} - 1{,}5$	$\frac{\frac{u_0}{d} + 3\pi}{\pi} - 1{,}5$

The graph in Fig. 3 clearly shows that condition (3) limits the number of possible perimeters of shear reinforcement to 4, 5, and 6, respectively, for the inner column, for the end column (and inner wall), and for the corner column (and end wall and wall corner). The values on the abscissa axis (Fig. 3) correspond to the distance of the perimeters of reinforcement from the support.

In all studies and graphs, the results for edge columns also apply to inner walls, and for corner columns, to edge walls and wall corners.

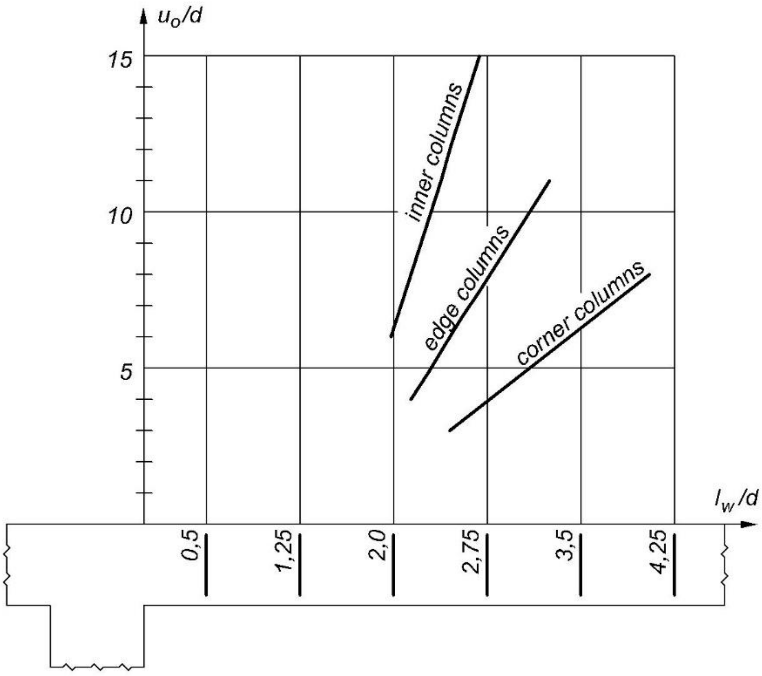

Fig. 3. Shear reinforcement region corresponding to maximum punching shear resistance

3.2 Investigation of compression diagonals

The condition for sufficiency of the compression diagonals limits the force that can pass through them to [1-5]:

$$\beta . V_{Ed} \leq V_{Rd,max} = 0{,}25 f_{cd} . u_0 . d \tag{7}$$

If both sides of inequality (7) are divided by $u_1 . d$, it follows:

$$\nu_{Ed} \leq 0{,}25 f_{cd} . \frac{u_0}{u_1} \tag{8}$$

The validity of (8) will be checked for the entire range of the investigated problem. For this, the maximum value of $\nu_{Ed} = 1{,}5\nu_{Rd,c}$ is accepted according to (3). Then (8) can be written:

$$\frac{1{,}5.\,\nu_{Rd,c}}{0{,}25 f_{cd}} \leq \frac{u_0/d}{u_1/d} \tag{9}$$

According to Table 1 and (9), the graph in Fig. 4 is constructed. The left side of the inequality is a constant value for a given concrete class and percentage of longitudinal reinforcement ρ_l. The curves corresponding to the right side of the inequality are constructed by consequently setting the ratio u_0/d for the different types of support.

It can be argued that for the entire range considered, the compression diagonals are not critical. Only for large values of $\rho_l \geq 1{,}3\%$ and low-class concrete ($C25/30$) checking the compression diagonals is necessary for inner columns with $u_0/d \leq 7{,}35$.

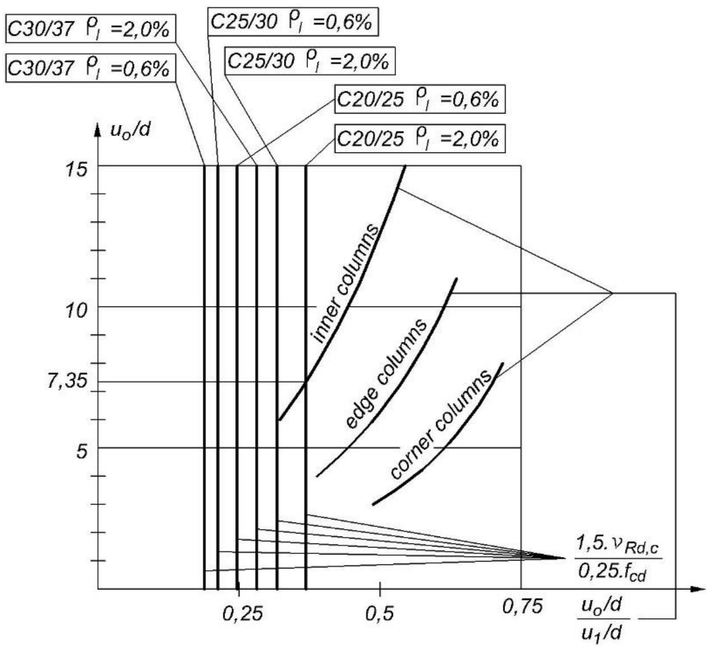

Fig. 4. Load-bearing capacity of compression diagonals

3.3 Cases for shear reinforcement regions

When placing a different number of perimeters of shear reinforcement around the supports, the achieved lengths r_{out} and respectively u_{out} can be determined. According to (4) and (1), the obtained relations u_{out}/u_1 will correspond to a specific degree of shear in the control perimeter v_{Ed} and should not be greater than $1{,}5.v_{Rd,c}$. These relations are systematized in Table 2.

Table 2. Ratio u_{out}/u_1, respectively $v_{Ed}/v_{Rd,c}$.

Number of shear reinforcement perimeters	Internal columns	Edge columns	Corner columns
two	$\frac{u_0/d + 5{,}5\pi}{u_0/d + 4\pi}$	$\frac{u_0/d + 2{,}75\pi}{u_0/d + 2\pi}$	$\frac{u_0/d + 1{,}375\pi}{u_0/d + \pi}$
three	$\frac{u_0/d + 7\pi}{u_0/d + 4\pi}$	$\frac{u_0/d + 3{,}5\pi}{u_0/d + 2\pi}$	$\frac{u_0/d + 1{,}75\pi}{u_0/d + \pi}$
four	$\frac{u_0/d + 8{,}5\pi}{u_0/d + 4\pi}$	$\frac{u_0/d + 4{,}25\pi}{u_0/d + 2\pi}$	$\frac{u_0/d + 2{,}125\pi}{u_0/d + \pi}$
five		$\frac{u_0/d + 5\pi}{u_0/d + 2\pi}$	$\frac{u_0/d + 2{,}5\pi}{u_0/d + \pi}$
six			$\frac{u_0/d + 2{,}875\pi}{u_0/d + \pi}$

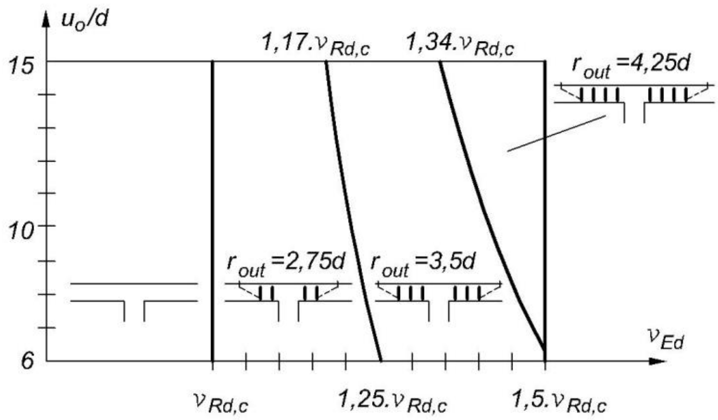

Fig. 5. Cases of shear reinforcement for inner columns

By successively setting u_o/d the graphics in Fig. 5, Fig. 6, and Fig. 7 are drawn. The graphics can be used to control the solutions at known stresses v_{Ed} in the control perimeter.

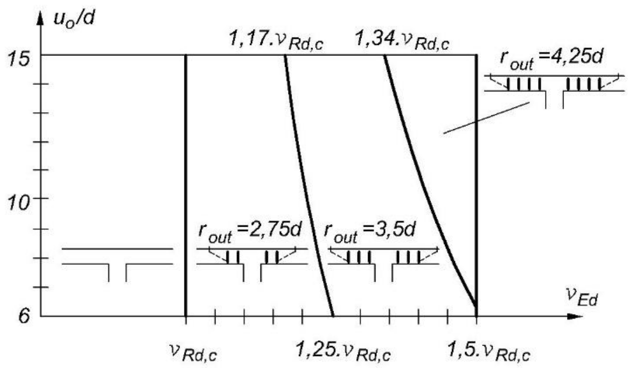

Fig. 6. Cases of shear reinforcement for edge columns and inner walls

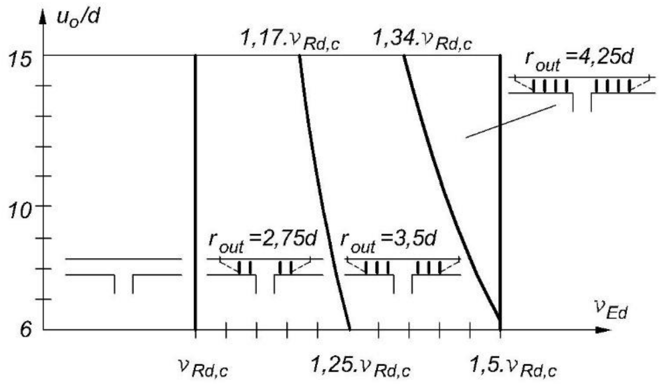

Fig. 7. Cases of shear reinforcement for corner columns and for edge walls and wall corners

3.4 Suitable diameter for shear reinforcement

According to Eurocode 2 [1, 2], the minimum area of one branch of transverse reinforcement is:

$$\frac{1{,}5.A_{sw,min}}{s_t.s_r} \geq \frac{0{,}08\sqrt{f_{ck}}}{f_{yk}} \tag{10}$$

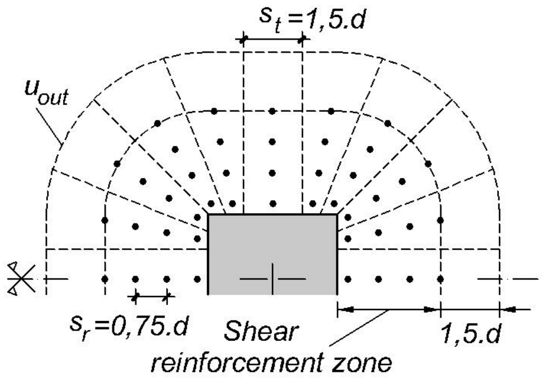

Fig. 8. Minimal shear reinforcement for each lamella

For a lamella with a width of $1{,}5d$, just like the maximum tangential distance s_t between the transverse reinforcement, it can be written:

$$A_{sw,min} \geq \frac{0{,}08\sqrt{f_{ck}}}{f_{yk}}.0{,}75.d^2 \tag{11}$$

Table 3. Minimum area of shear reinforcement

	$C20/25$	$C25/30$	$C30/37$
$d = 150mm$	$12{,}1\ mm^2$	$13{,}6\ mm^2$	$14{,}9\ mm^2$
$d = 200mm$	$21{,}5\ mm^2$	$24{,}1\ mm^2$	$26{,}4\ mm^2$
$d = 250mm$	$33{,}6\ mm^2$	$37{,}6\ mm^2$	$41{,}2\ mm^2$

Minimum reinforcement areas from Table 3 are much smaller than the real possible to place and traditional diameters [3, 4] for shear reinforcement with vertical branches. Even shear reinforcement with a diameter of ϕ8 is sufficient to satisfy (11) for the considered range of concrete classes and possible effective heights.

From (2) and according to Fig. 8, the relationship between the reduced contribution of the concrete in the considered lamella and the contribution of the inserted shear reinforcement from one branch in the same lamella is defined:

$$\frac{C}{R} = \frac{0{,}75.\nu_{Rd,c}}{\frac{1}{0{,}75.d^2}A_{sw,1}.f_{ywd,ef}} \tag{12}$$

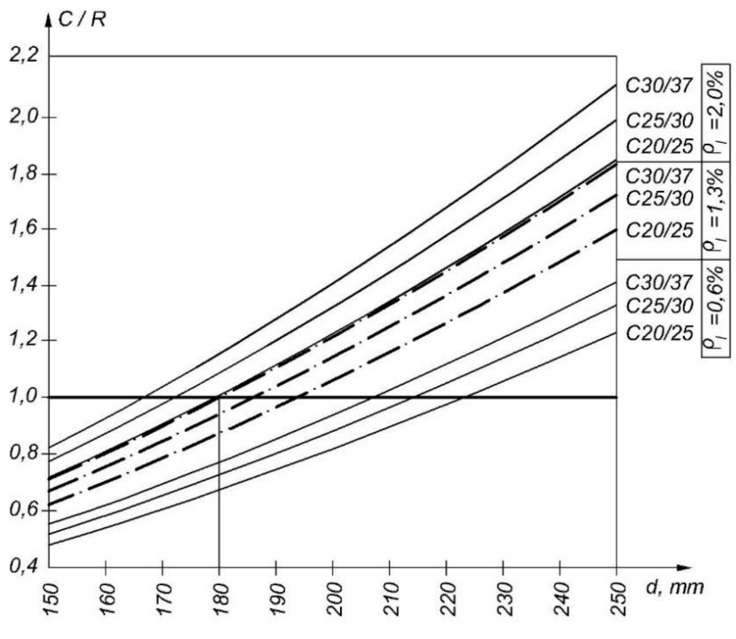

Fig. 9. Concrete contribution / Contribution of shear reinforcement ϕ8

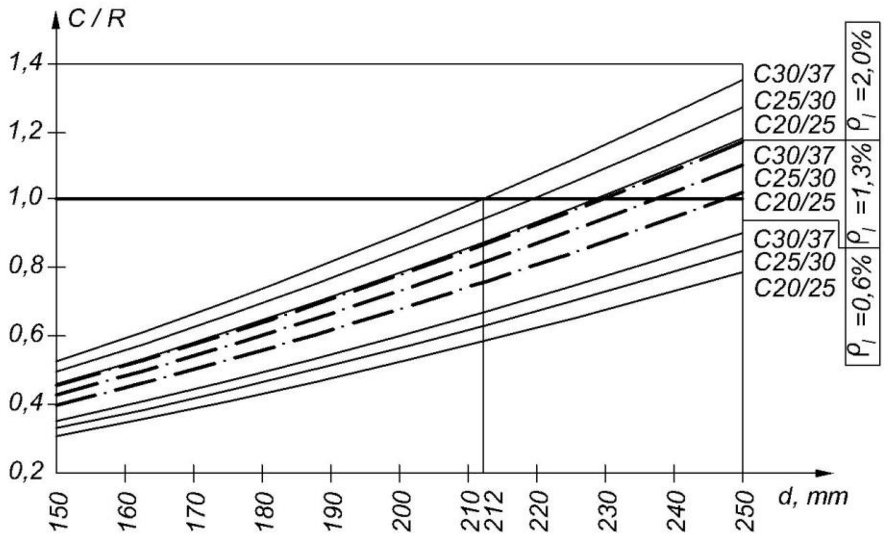

Fig. 10. Concrete contribution / Contribution of shear reinforcement ϕ10

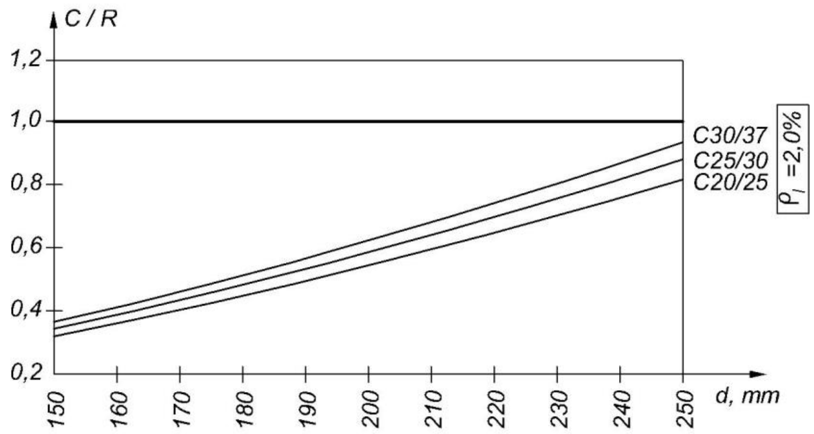

Fig. 11. Concrete contribution / Contribution of shear reinforcement $\phi 12$

For different combinations of concrete classes and reinforcement percentages of longitudinal reinforcement, the graphics in Fig. 9, Fig. 10, and Fig. 11 are drawn. Traditional diameters of shear reinforcement ($\phi 8$, $\phi 10$ and $\phi 12$) for floor slabs, are used. The effective height d and respectively $f_{ywd,ef}$ are changed in graphics.

If the values of C/R according to (12) are less than 1,0 the inserted shear reinforcement can absorb even a greater force than allowed.

From Fig. 9 for shear reinforcement $\phi 8$ it can be seen that a large part of the C/R curves are above the reference line of 1,0. It can be argued that with a diameter $\phi 8$ a greater force than allowed will be absorbed at smaller percentages of longitudinal reinforcement, i.e., at lower bending and respectively load. For example, at a percentage $\rho_l \leq 1{,}3\%$, regardless of the concrete class, shear reinforcement of $\phi 8$ will be sufficient for slabs with $d \leq 180mm$.

With a transverse reinforcement $\phi 10$ (Fig. 10), a greater than permissible force can be absorbed in slabs with $d \leq 212mm$, even when the reinforcement percentage is $\rho_l = 2\%$. Reducing the reinforcement percentage ρ_l extends the validity of the previous statement to larger effective heights.

With a transverse reinforcement $\phi 12$ (Fig. 11), a greater than permissible force can be absorbed in all examined slabs ($d \leq 250mm$) and at all reinforcement percentages ρ_l.

4. CONCLUSION

The introduced addition (3) limits the number of perimeters of shear reinforcement around the supports of flat slabs.

It has been proven that the study of compression diagonals makes sense in very rare cases.

Cases have been established in which the placement of traditional diameters for shear reinforcement at maximum constructive distances makes the design of punching unnecessary.

It turns out that when designing punching, it is only necessary to check the condition:

$$\nu_{Rd,c} \leq \nu_{Ed} \leq 1{,}5.\,\nu_{Rd,c} \tag{13}$$

and to arrange shear reinforcement in a certain number of perimeters (according to Fig.5, Fig.6, and Fig.7) and with an appropriate diameter (according to Fig.9, Fig.10, and Fig.11), while observing the design rules (Fig.2 and Fig.8).

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