SEARCHING FOR A COST-EFFECTIVE METHOD FOR ESTIMATING THE INITIAL AND MID-STRAIN SHEAR MODULUS OF SOILS BASED ON OEDOMETRIC TEST

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Abstract. Complex soil modeling has already been well established in the geotechnical engineering practice. Some of the soil parameters for the 21st-century constitutive models could be determined using classic laboratory tests; others, like the maximum shear modulus (G_0), shear wave velocity (v_s), reference shear strain parameter ($\gamma_{0.7}$) etc. needs specific methods for determination. The article focuses on searching for an alternative method that offers easier and cheaper determination of mid-strain as well as maximum shear modulus. One of them is to calculate it from the unloading curve of the compression (oedometric) test. Using a scientific approach and curve-fitting technique, it is possible to estimate the elastic behavior of soils to some extent. This study provides calculation results for the elastic behavior of the soils based on data from a few dozen soil samples. Some discussions on future ideas for determining the maximum shear modulus are also made.

Keywords: Small and mid-strain shear modulus; unloading soil stiffness; geotechnical laboratory testing; oedometer test;

1. INTRODUCTION

When dealing with old types of geological surveys and the modern way of geotechnical design (which is often the case in the Balkan region), there is always a lack of data regarding some specific soil parameters. Since the oedometer test is a classical soil test and its price is close to that of the “physical properties of the soil”, almost all the samples are subjected to it. In classical soil mechanics, we obtain the oedometer modulus and other consolidation parameters, like time of consolidation, over-consolidation ratio, etc. The modern soil mechanics has already established sufficient correlations with unloading, reloading modulus, and exponent (parameter) m. So we could say that every data obtained from the oedometer test could be correlated

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with more specific parameters. This study shows the first step for finding whether it is possible to obtain elastic soil behavior based on the unloading curve of the oedometer test data. G (E) $-$modulus reduction curve is well known in soil mechanics since 1991, where Atkinson (1991) [1] experimentally found that “strange” soil behavior. Later, Thomas Benz [2] presents his Hardening soil model with small-strain stiffness to address the nonlinear stiffness behavior of soils at small and very small strains, which traditional models often overlook. It introduces a strain-dependent stiffness framework, particularly focusing on how shear stiffness (G) decreases with increasing strain, capturing the initial elastic response more accurately. This model has been influential in enhancing numerical simulations in geotechnical engineering, especially in capturing ground behavior under low-amplitude loading conditions such as settlement, anchor walls, seismic, or machine-induced vibrations. G modulus reduction curves are mainly obtained through laboratory tests such as resonant column or cyclic triaxial tests, which apply small, controlled strains to soil samples. The shear modulus (G) is measured at various strain levels, showing how it decreases with increasing strain. These measurements are normalized by dividing by the initial shear modulus (G_0) to create the G/G_0 vs. shear strain curve. This study focuses on proving that to proof it is possible to obtain some G modulus reduction curve or at least part of it from classical laboratory apparatus such as the standard oedometer test. [3], [4]. For that case, 60 soil samples from different types of Bulgarian soil are used.

2. SOIL SAMPLES

The Balkan region is situated in the southeast of Europe. It is characterized by normally consolidated quaternary clays, silts, and sand, and mostly a mix of them. This study uses a soil library that contains data for silty or sandy clays and clayed silty sands sampled from different parts of quaternary valleys (Balkan region). The plasticity index varies from 0 to 60, and the percentage of Sand particles is between 0 and 84 %. Since the soil samples are saturated, other physical or plastic parameters don’t affect the G modulus as much, which is why they are not considered in the study. The test procedure is well known from the oedometric test standards [4] [5]: saturation, doubling the loading pressure with every step; time interval for the steps - 24 hours. The most relevant for the study unloading steps are done every 2 hours following the same pressure values as the loading, but in the opposite direction. When derived the data were not expected to take part in such of study; that’s why their accuracy is not distinguished, so the significance of a single sample value is not relevant for any conclusions. The vertical displacements during the test were recorded using a standard mechanical dial gauge, which provides reliable and precise measurements of settlement over time. Data acquisition was carried out manually, with particular attention given to capturing the displacement readings immediately before each unloading phase. This approach ensured that the most critical data points, reflecting the soil's deformation behavior under each loading stage, were accurately documented without unnecessary intermediate readings.

3. CALCULATION PROCEDURE

Estimating the modulus from the unloading portion of the oedometer curve presents certain challenges (Fig. 1) [6], as the test data were not originally intended for such evaluation.

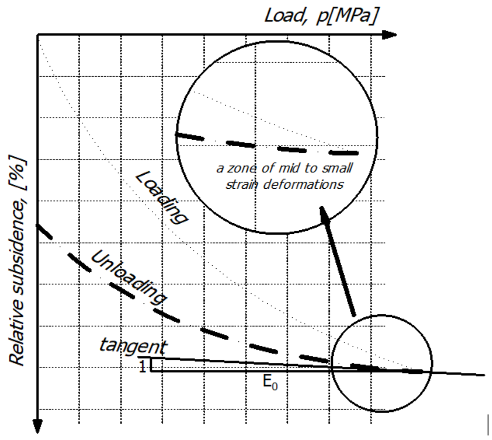

Fig. 1. Concept for estimation of mid and small-strain soil behavior based on oedometric tests [1].

The unloading path typically reflects elastic rebound; however, its interpretation can be complicated by factors such as time-dependent effects, incomplete recovery, and prior plastic deformations. As a result, deriving a reliable elastic modulus from this segment requires careful analysis and may not strictly follow standard procedures. In this study, the same analytical procedure was systematically applied to several dozen soil specimens to ensure consistency in the evaluation of unloading behavior. Data sets that exhibited inconsistencies or significant deviations from the expected trend were excluded from further analysis to maintain the reliability of the results. From the accepted test series, stress–strain data corresponding to the unloading phase were extracted, and the corresponding $\sigma-\varepsilon$ diagrams were constructed to assess the soil's elastic response. Subsequently, the extracted unloading data were fitted using a shape-preserving piecewise cubic interpolation in order to obtain a smooth approximation suitable for derivative analysis. The derivative of this fitted curve, representing the tangent modulus (E-modulus), was then calculated at several specific points along the unloading segment. Particular attention was given to evaluating the modulus at the onset of unloading, where the stress-strain response begins to exhibit elastic recovery. It should be noted, however, that this region does not strictly fall within the small-strain domain, as the values are derived from the interpolated curve based on the first three measured points of the unloading path (Fig. 2).

The deformation levels corresponding to the evaluated points along the unloading curve were found to lie approximately within the strain range of 0.0005 to 0.001. These values clearly exceed the conventional threshold for small-strain behavior, typically considered to be below 0.0001, and are more appropriately categorized within the mid-strain range. Despite not capturing the true initial elastic stiffness of the soil, these strain levels still provide meaningful insights into the material's stiffness degradation characteristics during unloading. Given that over 60 soil samples were analyzed using this approach, the resulting data allowed for the identification of consistent trends in stiffness behavior. From this, approximate modulus reduction curves—reflecting both the reduction in modulus (E) and the corresponding shear modulus (G) with increasing strain were derived and are presented in Fig. 3. Although the method does not fully isolate small-strain elastic properties, it still enables valuable comparative analysis across different soil types and conditions.

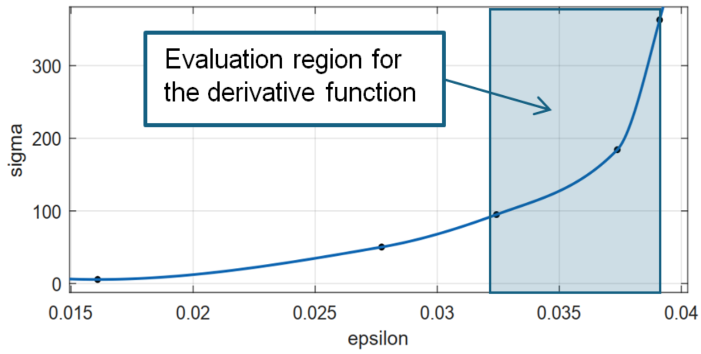

Fig. 2. Pricewise shape preserving cubic fit for the unloading $\sigma - \varepsilon$ diagram, (σ varies between 0 and 400 kPa).

The transition between elastic modulus and shear modulus could be done using the classical elastic theory:

$$G = \frac{E}{2(1+\upsilon)}, \quad (1)$$

where E is the elastic modulus, ν – Poisson ratio.

The Poisson ratio of the soils tends to be higher at lower strain [7]. In the range of mid strains, we could obtain a mid Poisson ratio of 0.35.

When analyzing soil behavior under triaxial or oedometer loading conditions, it is essential to establish a reliable relationship between axial (longitudinal) and shear deformation components. This relationship becomes particularly important when attempting to interpret moduli obtained from one stress path (e.g., uniaxial compression) in terms of their relevance to shear behavior. For that purpose, an equivalent theory based on the von Mises failure criterion (i.e., $p - q$ vs $\sigma - \tau$ transition) could be used [8]. When analyzing soil behavior under triaxial or oedometer loading conditions, it is essential to establish a reliable relationship between axial (longitudinal) and shear deformation components. This relationship becomes particularly important when attempting to interpret moduli, based on that:

$$\gamma_s = \varepsilon_q = \frac{2}{3}|\varepsilon_1 - \varepsilon_3|. \tag{2}$$

With $\varepsilon_3 = 0$ for oedometer condition, the shear strain has the following value:

$$\gamma_s = \frac{2}{3}|\varepsilon_1| \tag{3}$$

4. OBTAINED RESULTS

The results are presented in two ways: G-modulus reduction curve (Fig. 3) and correlation for the obtained G modulus with respect to other soil sample parameters.

Most of the tested soil samples show a consistent trend of higher shear modulus at lower strain levels. This behavior aligns with the expected non-linear stiffness increase as strain decreases. However, the minimum strain reached in the tests is approximately 0.0008. This is above the threshold typically associated with small-strain elastic behavior, generally considered to be below 0.0001. Therefore, the results do not capture the true elastic shear modulus. The median shear modulus across all samples is around 38,000 kPa. This value is significantly lower than the estimated average small-strain shear modulus of 65,000 kPa obtained from specialized laboratory testing. The difference is largely due to the limitations of the strain range. Additionally, the dataset includes a wide variety of soil types with differing physical and mechanical properties. As a result, the standard deviation of the measured shear modulus values is high, reflecting the heterogeneity of the sample set.

Complete data on the physical and plastic properties of all soil samples were not available for this study. Furthermore, many common empirical correlations related to shear modulus (G) are not directly applicable due to the fully saturated condition of the specimens. Saturation affects soil stiffness and can mask the influence of factors like plasticity or structure. However, one relationship remains partially valid—the influence of particle size distribution, particularly the sand content. It is well-established that soils with higher sand content typically exhibit higher shear modulus values (Fig. 4).

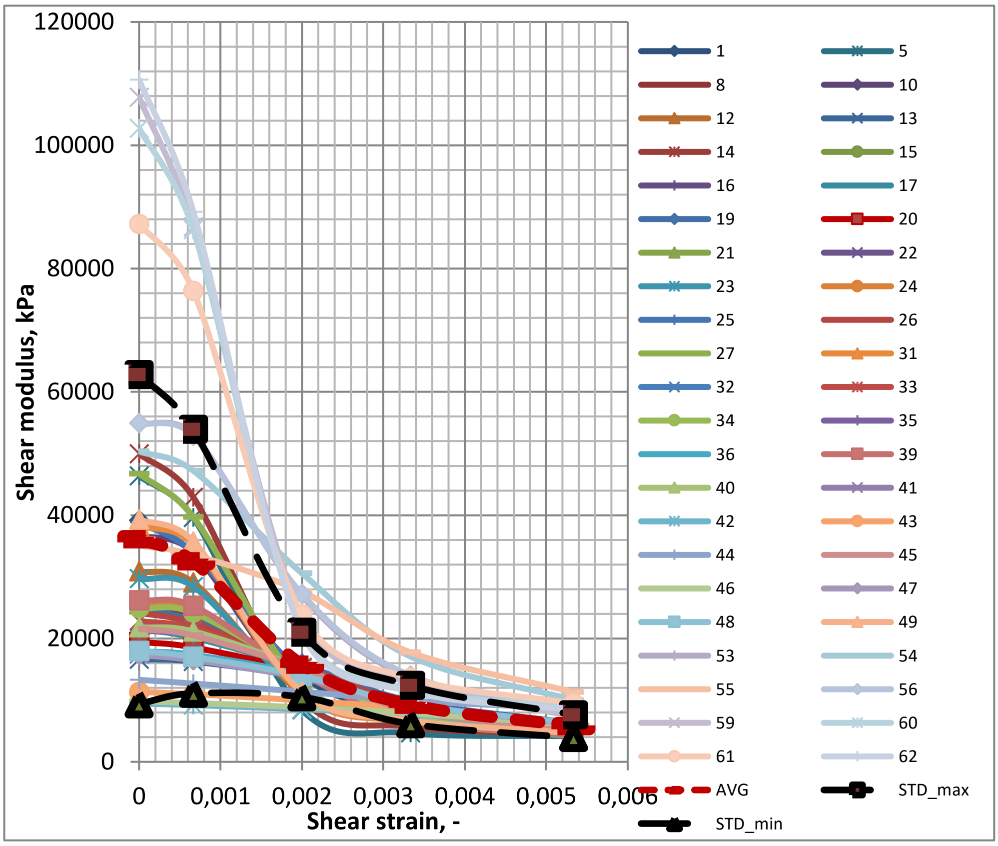

Fig. 3. G modulus reduction curve (estimated from oedometer test unloading data)

This trend is also observed in the current dataset. Nevertheless, most of the samples in this study have low sand content, as the majority are clay-rich. As a result, the data are concentrated in the lower sand percentage range, limiting the strength of any observed correlation. A simple linear regression gives the empirical expression

$$G \approx \sim 180.(\%sand) + 26000, kPa \tag{4}$$

with $R^2 \approx 0.59$, indicating a moderate, statistically significant positive dependence between sand fraction and mid‑strain shear modulus. The moderate strength of the correlation reflects two competing factors: the dominance of clay‑rich samples (sand < 15 %), which compresses the predictor range; the saturated test conditions, which partly obscure the heterogeneity in interparticle contacts that would otherwise enhance stiffness in sandier soils.

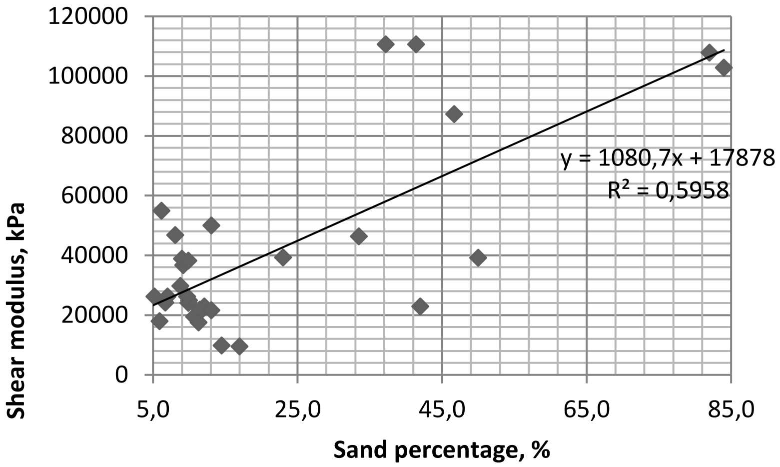

Fig. 4. Shear modulus at mid-strain vs sand percentage of the soils

Another potential correlation could be explored with respect to the Plasticity Index (Fig. 5), although only a general trend was observed.

A dedicated statistical assessment clarifies the relationship between the mid‑strain shear modulus (G) and the plasticity index (Ip) depicted in Fig. 5.

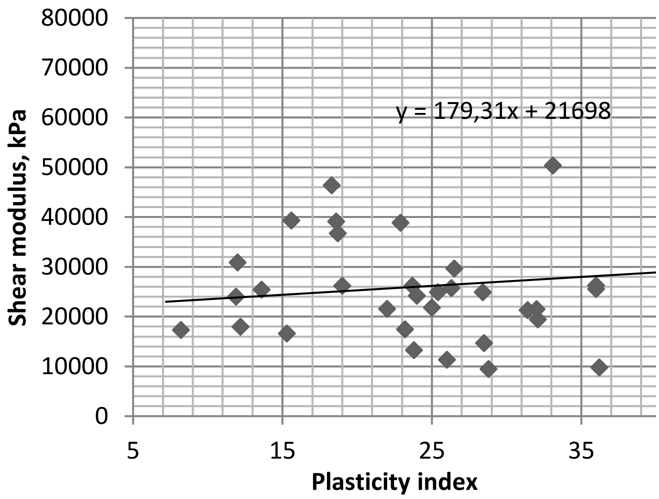

Fig. 5. Shear modulus at mid-strain vs plasticity index

A simple least‑squares linear regression produced a coefficient of determination $R^2 \approx 0.18$, while a non‑parametric Spearman rank‑correlation analysis yielded

$\rho \approx -0.21$. Both measures indicate a weak and statistically insignificant trend. In practical terms, within the investigated mid-strain domain, the shear modulus does not exhibit a systematic dependence on Ip.

The variation in soil types, particularly in terms of their plasticity, further complicates the interpretation of any direct correlation.

5. CONCLUSION

This study does not provide a sufficiently robust foundation for establishing a practical correlation between the shear modulus (G) and its reduction curve derived from the unloading portion of standard oedometer tests. The data sampling was not conducted with the expectation of achieving high precision, and as such, the study primarily captures the mid-strain shear modulus.

This results in a relatively high standard deviation in the calculated data. While it is possible to segregate and analyze different soil types individually, more precise measurements are necessary to yield more reliable and consistent results. Considering that linear voltage differential transducers with up to 20 times greater precision are possible [9], capable of measuring at intervals as short as every second, significantly more accurate results could be achieved.

When evaluating mid- and small-strain stiffness, it is crucial that the data collection process incorporates very small increments along the unloading curve. Ideally, these steps should be as small as 2 to 5 kPa per step to accurately capture the material's response. This level of resolution is particularly important as it allows for a more appropriate analysis of the soil's stiffness behavior, especially within the small-strain range. Small incremental steps are essential for reaching the elastic region, typically considered to be between 10^{-4} and 10^{-5} measured in strain levels. By using such fine gradations, it becomes possible to detect subtle variations in stiffness that might otherwise be overlooked. The impact of these small steps on the dataset can be significant, as they enable a more precise characterization of the soil's elastic properties and its response under low-amplitude loading conditions, which are critical for an accurate modeling of soil behavior in practical engineering applications.

Having in mind all said above, we could conclude the following. Although the study doesn't reach the small-strain domain, it successfully characterizes mid-strain stiffness for a wide range of soils using unloading curves, which is less common in typical oedometer analyses. With over 60 samples, the study provides a substantial dataset that captures variability across different soil types, offering valuable reference points for future research on modulus-strain behavior. By highlighting the limitations of standard measurement techniques and pointing toward more advanced technologies (e.g., high-precision LVDTs), the study encourages the adoption of improved instrumentation for soil stiffness testing. While practical correlations could not be established due to measurement limitations, the study sets a methodological basis for future research to build upon with more precise equipment and refined protocols. The study implicitly emphasizes the importance of step size and sampling frequency when designing laboratory tests aimed at capturing small- and mid-strain soil behavior.

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