ADAPTIVE COMBINED HIGH-PASS AND MAINS INTERFERENCE REJECTION FILTER

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Abstract. The paper presents an alternative approach for high-pass filtering of ECG and simultaneous removal of mains interference. The approach is based on the summation of several filters having zeros for the mains and zero frequencies. A program is compiled that calculates the coefficients of the combined filter depending on the set cut-off frequency for the high-pass filtering, the sampling rate, and the frequency of the mains interference. The combined filter is adaptive to the mains interference's frequency deviation. The implementation of the filtering is carried out in three consecutive steps: i – the mains interference is subtracted from the ECG signal; ii – the change in the mains frequency relative to its rated value is determined; iii – the coefficients of the combined filter are recalculated. The mathematical basis for synthesizing the combined filter is presented. Tests performed with artificial and real ECG signals and disturbances prove that the combined filter is able to remove very intense baseline drift and completely suppress mains interferences with minimal impact on the ECG signal shape.

Keywords: Digital filtering, ECG signal, Baseline drift, mains interference.

1. INTRODUCTION

Two of the main disturbances present in equipment processing ECG signals are baseline drift and mains interference. Considerable research has focused on designing digital filters to correct baseline drift and remove mains interference without introducing distortions to the shape of the ECG signals. There are different drift suppression approaches, based both on the classic high-pass FIR and IIR filters using the Hamming window [1], as well as specialized procedures, such as High-pass filtering using QRS elimination [2], optimized bi-directional high-frequency filtering [3],

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adaptive filtering by wavelet transforms [4], etc. Multiple digital filters are constructed to eliminate mains interference. The most effective are adaptive filters [5], bi-directional filtering [6], subtraction procedure [7], etc.

Simultaneous suppression of baseline drift and powerline interference using a non-linear filter bank is proposed in [8].

The introduction in [9] combined a high-pass and powerline interference rejecter filter (so-called FilterDxN) is characterized by simplicity and sufficient efficiency. The principle of its operation is based on the averaging of N samples with a predefined distance D between them. The filter has zeros for the multiples of the mains frequency, thanks to which it also suppresses the harmonics of the mains interference. The validation of the FilterDxN (variant Filted10x17) in both clinical and test ECG databases is reported in [10]. The FilterDxN thus constructed is not adapted to operate in the non-multiplicity between the frequency of the mains and the sampling rate.

The present research aims to create a similar filter, retaining the main advantages of FilterDxN, while making it independent of sampling rate and mains frequency. At the same time, the aim is to adapt it to deviation in the mains interference frequency.

2. MATERIAL AND METHODS

In [11], a methodology is proposed for synthesizing a complex linearity criterion, which is an elementary filter with zeros for both the mains frequency and near-zero frequencies. On the basis of this methodology, a combined high-pass and notch filter for mains frequency is proposed. Its construction is carried out by following the steps:

1. A complex basal **C**-filter summing *d* number of filter pairs is synthesized. The **C1**- and **C2**-filters contained in each pair are summed in a ratio determined by the factors *c*. The factor *c* is set so that the **C**-filter has a minimum at the mains frequency. The transfer function of the **C**-filter in the z-plane is given by the equation

$$C = \frac{1}{d}\sum_{j=1}^{d}\left[c_j . C1_j + \left(1 - c_j\right). C2_j\right], \qquad \left| \begin{array}{l} C1_j = \dfrac{-z^{-n_j-1} + 2 - z^{n_j+1}}{2} \\ C2_j = \dfrac{-z^{-n_j} + 2 - z^{n_j}}{2} \end{array} \right. ,$$

$$c_j = \frac{1}{1 - \dfrac{\left(n_j + 1\right) \cdot cos\dfrac{\pi . F . \left(n_j + 1\right)}{Q} \cdot sin\dfrac{\pi . F . \left(n_j + 1\right)}{Q}}{n_j \cdot cos\dfrac{\pi . F . n_j}{Q} \cdot sin\dfrac{\pi . F . n_j}{Q}}}, \quad n_j = int\left(j \cdot \frac{Q}{F}\right). \tag{1}$$

The parameter nj is the rounded integer number of samples in j number of periods of the mains frequency. The designations here and below are: F is the rated value of the mains interference frequency; Q is the sampling rate. The transfer coefficient of the complex **C**-filter at the frequency of the mains interference is

$$C_F = \frac{2}{d}\sum_{j=1}^{d}\left[c_j \cdot sin^2\frac{\pi.F.\left(n_j+1\right)}{Q} + \left(1-c_j\right)\cdot sin^2\frac{\pi.F.n_j}{Q}\right] \tag{2}$$

2. An auxiliary A-filter is synthesized, summing two filters A1 and A2 in a ratio determined by the factor a. The factor a is set so that the A-filter acquires a transfer coefficient for the frequency of the mains interference equal to C_F . The transfer characteristic of the A-filter in the z-plane is given by

$$A = \frac{[a.A1+(1-a).A2].C_F}{A_F}, \quad \left| \begin{array}{l} A1 = \dfrac{z^{-m-1}-2+z^{m+1}}{4} \\ A2 = \dfrac{z^{-m}-2+z^{m}}{4} \end{array} \right. ,$$

$$a = \frac{1}{1-\dfrac{(m+1)\cdot cos\dfrac{\pi.F.(m+1)}{Q}\cdot sin\dfrac{\pi.F.(m+1)}{Q}}{m\cdot cos\dfrac{\pi.F.m}{Q}\cdot sin\dfrac{\pi.F.m}{Q}}}, \quad m = int\left(\frac{Q}{2.F}\right), \tag{3}$$

$$A_F = -\left[a\cdot sin^2\frac{\pi.F.(m+1)}{Q} + (1-a)\cdot sin^2\frac{\pi.F.m}{Q}\right]$$

The parameter m is the number of half-period samples of the mains frequency, rounded to less than or equal to an integer. The factor A_F represents the transfer coefficient of the auxiliary A-filter at the frequency of the mains interference;

3. The auxiliary A-filter is subtracted from the C-filter, thus resulting in the R-filter

$$\mathrm{R} = \mathrm{C} - \mathrm{A} \tag{4}$$

having zeros for both the mains interference frequency and the zero frequency;

4. A unit vector $1 = z^0$ is added to the auxiliary A-filter, and the convolution operation is applied between the resulting sum filter and the R-filter to obtain the combined high-pass and notch G-filter

$$\mathrm{G} = \mathrm{R}.(\mathrm{A}+1). \tag{5}$$

Synthesizing a combined high-pass and notch filter for the mains frequency is illustrated in Fig. 1, where the first subplot shows the frequency responses of the C-, A-, R-, and G-filters, calculated respectively according to (1-5). The second subplot contains the phase response of the G-filter. The characteristics are obtained at a sampling rate Q = 250 Hz and a mains interference frequency F = 60 Hz. The methodology is illustrated just for one pair of C1- and C2-filters ($d = 1$).

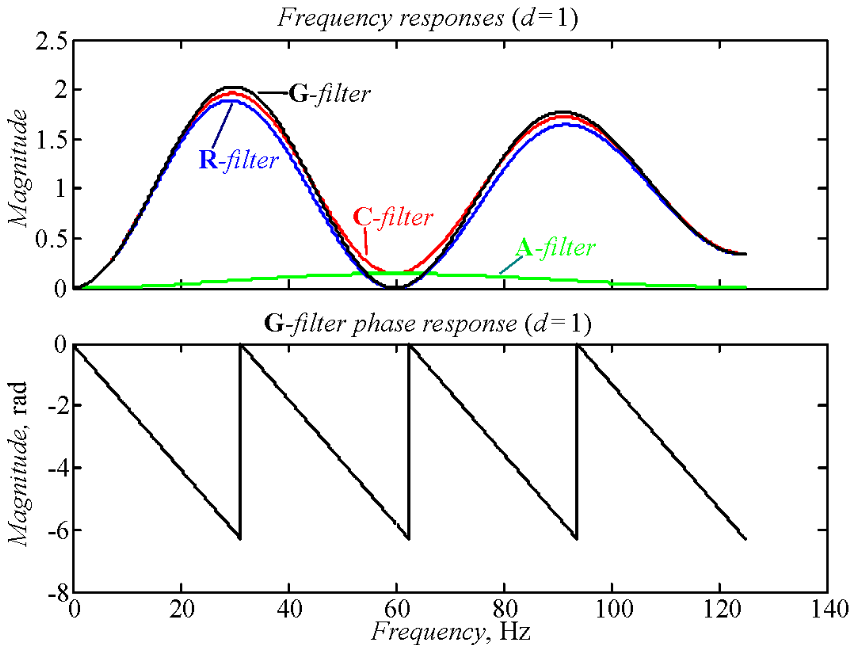

Fig. 1. Methodology for synthesizing a combined high-pass and a mains notch filter.

Figure 2 shows the frequency responses of the combined filter for cases of sampling rate Q = 250 Hz and the two mainly used mains frequencies – 60 and 50 Hz.

When there is an integer odd multiplicity between the sampling rate and the mains frequency, the combined filter suppresses the harmonics of the mains interference (see lower subplot on Fig. 2, where at Q = 250 Hz and F = 50 Hz, the integer multiplicity is 5 – in this case, for F_h = 1 Hz Filter5x34 is obtained). This property is not preserved in the absence of multiplicity or in the violation of multiplicity due to deviation of the mains frequency.

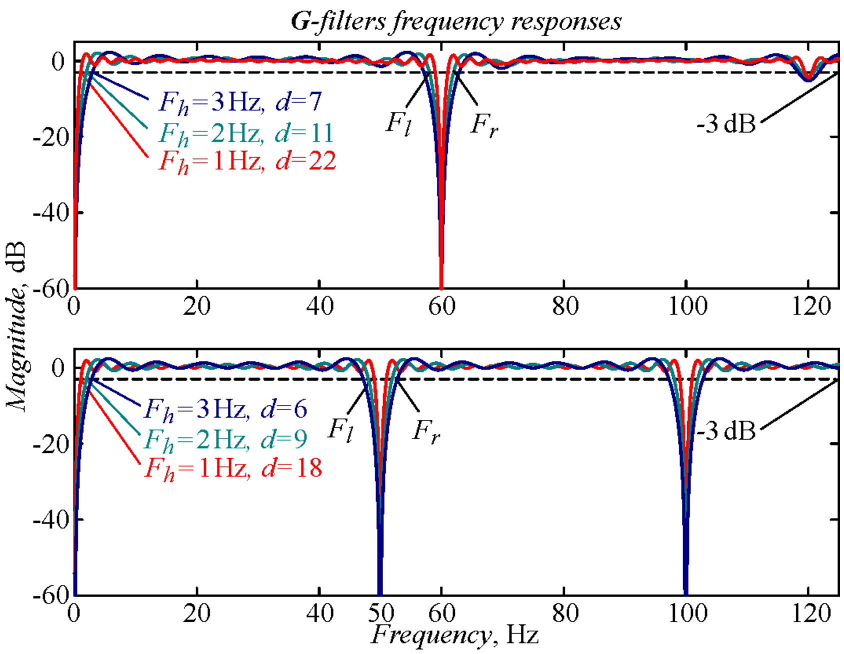

Fig. 2. Frequency responses of a combined high-pass and notch filter for mains interferences 60 Hz (upper subplot) and 50 Hz (lower subplot).

A feature of the combined filter is that the rejection band for the mains frequency $F_r - F_l$ is twice as wide as the cut-off frequency F_h, i.e. $F_r - F_l = 2.F_h$, therefore, the quality factor of the filter for the mains frequency is $Q_F = 2.F_h/F$.

A more detailed consideration of the frequency response of the combined filter at Q = 250 Hz, F = 50 Hz and F_h = 1 Hz is presented in Fig. 3.

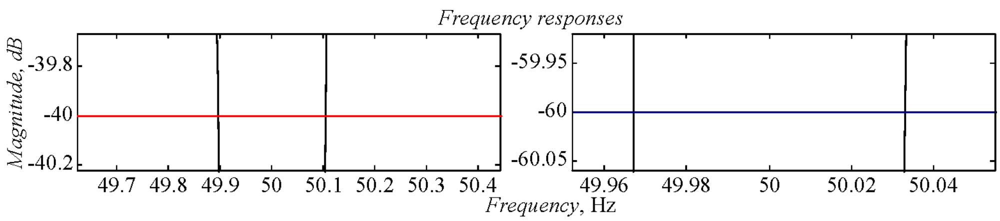

Fig. 3. Zoomed frequency responses at levels -40 dB (left subplot) and -60 dB (right subplot).

As can be seen, the combined filter provides suppression of -40 dB within ±0.1 Hz and -60 dB within ±0.03 Hz relative to the mains frequency. This means that if it is necessary to provide a suppression of -40 dB within ±1 Hz, the G-filter must change coefficients at least 10 times with a step of 0.02 Hz. Accordingly, for the suppression of -60 dB filter must be changed coefficients minimum of 30 times in a step of 0.06 Hz. Analogous conclusions can be drawn from the analysis of the filter at F = 60 Hz. Based on this analysis, it is proposed to divide the possible range d_F of the frequency variations of the mains into segments and to apply a separate G-filter for each segment.

3. SYNTHESIS OF FILTER COEFFICIENTS

A program was developed in the MATLAB environment, which calculates the values of the combined high-pass and notch filter coefficients, according to formulas (1-5). Input parameters in the calculations are the sampling rate Q, the rated value of the mains frequency F, the cut-off frequency F_h and the possible range d_F of the mains frequency deviation. The program calculates the coefficients of S number of G-filters in the range $[F - d_F/2 \ldots F + d_F/2]$, using the method of successive approximations. At the beginning, the value of the parameter $d = 1$ is set and by incrementing d, formulas (1-5) are executed cyclically, until a cut-off smaller than or equal to the preset F_h is obtained. The calculated coefficients are stored in a matrix G[S×L], where $L = 2M + 1$ is the number of terms in G, and M is the number of the central term.

For the case Q = 250 Hz, Table 1 presents the values of the parameter d and the number of coefficients L, depending on the set cut-off frequency F_h. Graphically, dependencies are presented in Fig. 3.

Table 1. Dependences of the parameters d and L on F_h

F_h, Hz	d (F = 50 Hz)	d (F = 60 Hz)	L (F = 50 Hz)	L (F = 60 Hz)
0,5	37	45	279	383
0,75	25	30	259	259
1,0	18	22	189	191
1,25	15	18	159	159
1,5	12	15	129	133
1,75	10	13	109	117
2,0	9	11	99	99
2,25	8	10	89	91
2,5	7	9	79	83
2,75	6	8	69	75
3,0	6	7	69	67

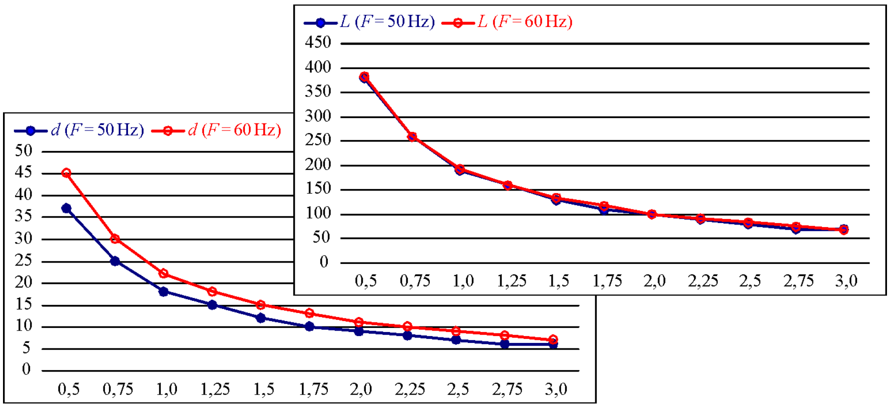

Fig. 4 Dependence of the number d of averaged filters and the length L of the combined filter on a set cut-off.

Table 2 shows the dependence of the number of coefficients L of the combined filter on the sampling frequency at a cutoff frequency F_h = 1 Hz.

Table 2. The dependence of the number of coefficients L on the sampling rate

Q, Hz	200	250	300	350	400	450	500	550	600	650	700
F = 50 Hz	153	180	227	263	301	337	375	411	449	485	523
F = 60 Hz	153	191	229	265	303	341	379	415	455	491	527

4. COMBINED FILTER IMPLEMENTATION

The implementation of the filter is performed by the computational procedure

$$Y_i = \sum_{l=1}^{L} I_{i-M+l} . G_{s,l} \tag{6}$$

Here and further, Y is the vector of the filtered signal, I is the vector of the contaminated ECG signal, and index "i" marks the ongoing value. Coefficients $G_{s,l}$ are elements of the matrix G[S×L]. The index "l" is an integer in the range [1…L], which determines the matrix's column. The parameter "s" is an integer in the range [1…S], which determines the matrix's row. The determination of the parameter "s" is performed in the following sequence:

1. A B-filter $B = 1 - G$is constructed, where $1 = z^0$, through which the mains interference is removed from the input signal. The implementation of the B-filter is carried out through the formula

$$B_i = Y_i - I_i \tag{7}$$

2. An estimator filter $E = [1 - b_i.z^{-q} + b_i.z^{-3q} - z^{4q}]/4$ is applied to samples B_i. In addition to the extracted mains interference, the resulting vector B also contains low-frequency components. Applying the thus constructed E-filter zeros the low-frequency components and allows for estimating the value of the frequency of the mains frequency to be estimate. The frequency response of filter B for the case Q = 250 Hz and F = 60 Hz is shown in Fig. 5.

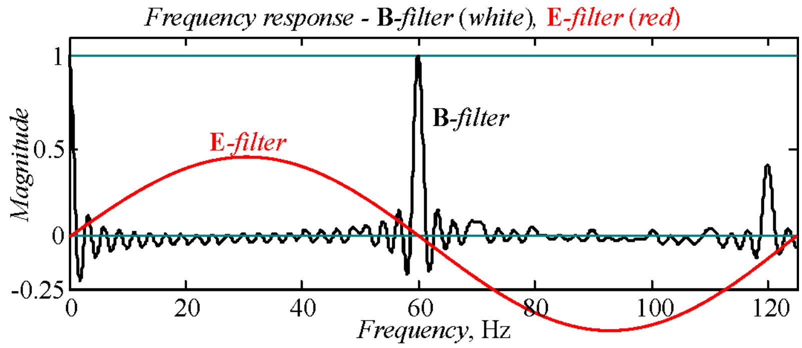

Fig. 5. Frequency responses of the removed filter B and the estimator filter E.

By setting E = 0, the value of b_i is calculated at which the filter E retains zero at the current frequency of the mains interference

$$b_i = \frac{B_i - B_{i-4q}}{B_{i-q} - B_{i-3q}}, \qquad q = int\left(\frac{Q}{4.F}\right) \tag{8}$$

3. The coefficient b_i is limited by the speed b_{spd} and boundaries $b_{min} < b_i < b_{max}$ of the mains frequency variation through the equation (MATLAB).

```
b_spd = (b_max-b_min)/(Q/q);
if b(i)-b(i-1)>b_spd; b(i)=b(i-1)+b_spd; end;
if b(i)-b(i-1)<-b_spd; b(i)=b(i-1)-b_spd; end;
if b(i)>b_max; b(i)=b_max; end;
if b(i)<b_min; b(i)=b_min; end;
```
(9)

The equivalences in (9) are: `b(i)` $\equiv b_i$; `b_spd` $\equiv b_{spd}$; `b_min` $\equiv b_{min}$; `b_max` $\equiv b_{max}$.

4. The value of the row s of the matrix G[S×L] is calculated by the formula

$$s = 1 + int\ [(b_i - b_{max})\ \cdot\ K_b],\quad K_b = \frac{S-1}{b_{min} - b_{max}}$$

$$b_{max} = 2.cos\frac{2.\pi.q.(F - F_h)}{Q} \tag{10}$$

$$b_{min} = 2.cos\frac{2.\pi.q.(F + F_h)}{Q}$$

The implementation of the combined filter is illustrated in Fig. 6. Constants b_{min}, b_{max} and K_b are preliminary calculated. The coefficient b is filtered in the function block Φ.

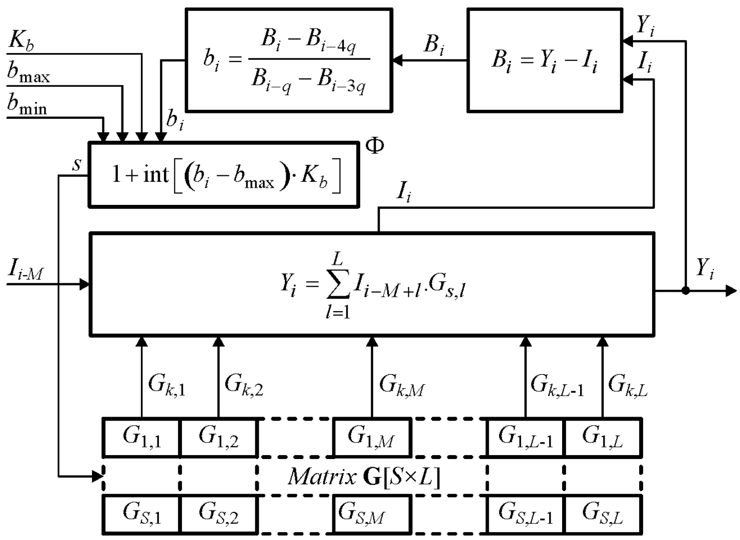

Fig. 6. Combined high-pass and adaptive notch filter flow chart.

5. EXPERIMENTS

In the test experiments with the combined high-pass and notch filter, 16 seconds of artificially synthesized signals with a sampling rate Q = 250 Hz are used:

- Mains interference with an amplitude of 1 mV (p-p) and a frequency with rated value F = 60 Hz, deviating within ±0.5 Hz (d_F = 1 Hz);
- Low-frequency baseline drift with an amplitude of 1 mV (p-p) and changing frequency during recording from 0 to 0.4 Hz;
- A simulated signal ECGprobe, reproducing a “normal” form of a signal;
- Episodes of real ECG signals from the AHA databases with interferences.

Figure 7 shows the results when filtering the synthesized mains interference. The first subplot contains pure interference, and the second one – residual interference after filtering. The third subplot shows the change of the mains frequency and the calculated parameter *s* during the filtering process.

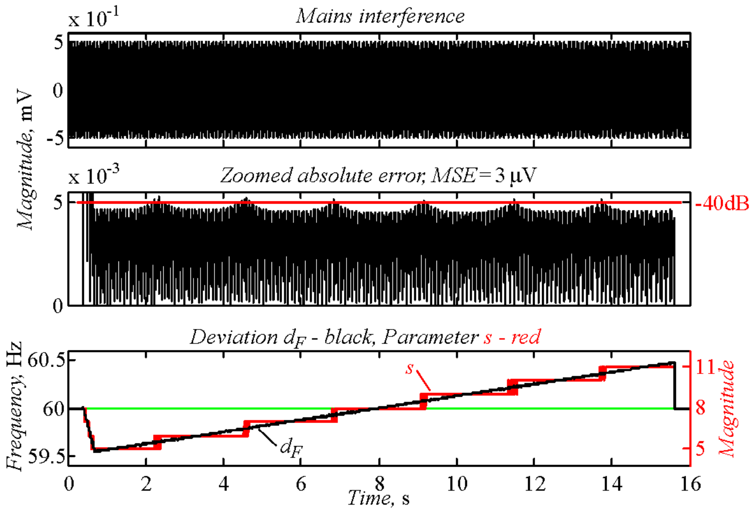

Fig. 7. Experiment with varying mains frequency.

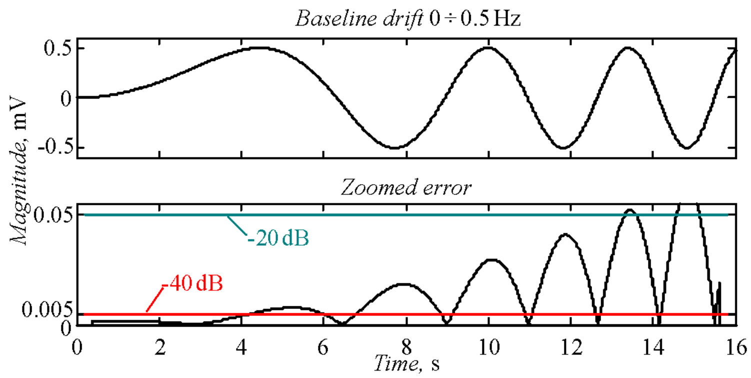

Fig. 8. Experiment with varying baseline drift.

Figure 8 illustrates baseline drift suppression. The test drift is shown on the first subplot. The second one contains the absolute error after filtering. It can be observed that –20 dB of attenuation is achieved at a frequency of 0.3 Hz, and –40 dB – at a frequency of 0.08 Hz.

The result of applying the combined filter to the ECGprobe test signal is shown in Fig. 9. The first subplot contains the test signal with mains interference and low-pass drift added, and the second subplot shows the filtered signal. The third subplot shows the errors introduced into the filtered signal by the applied filters. The error was calculated as the difference between the pure ECG probe signal and the signal after filtering. The error shown in red is when the filtering is applied without compensating for the mains frequency deviation.

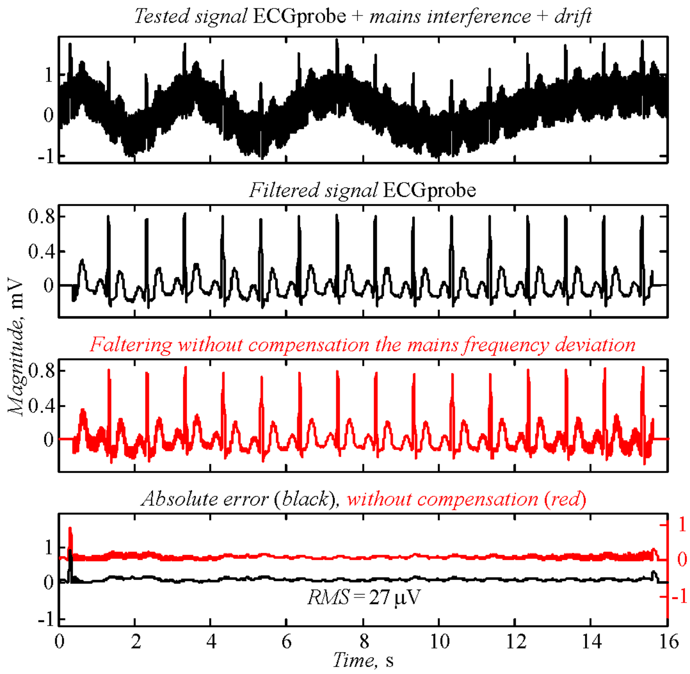

Fig. 9. Experiment with the ECGprobe test signal.

The following Fig. 4 shows the results of applying the combined high-pass and notch filter (1 Hz cut-off) to the ECGprobe test signal. The first subplot of each case contains the test signal with the interference added, and the second subplot shows the filtered signal. The third subplot shows the involved fault in the filtered signal by the applied filters. The involved fault is calculated as the difference between the pure ECG probe signal and the signal after processing. For comparison, filtering was additionally performed with a reference filter. As such, the convolution between a 1st-order digital high-pass filter and a modified comb filter is used.

The performance of the proposed filter with real ECG recordings contaminated by significant baseline drift, tremor disturbances and small 60 Hz mains interferences is shown in Fig. 10. The first subplot of each illustration contains the real ECG recordings, the second subplot shows the filtered signal and the third subplot shows the removed components, as a difference between the real and the signal after processing.

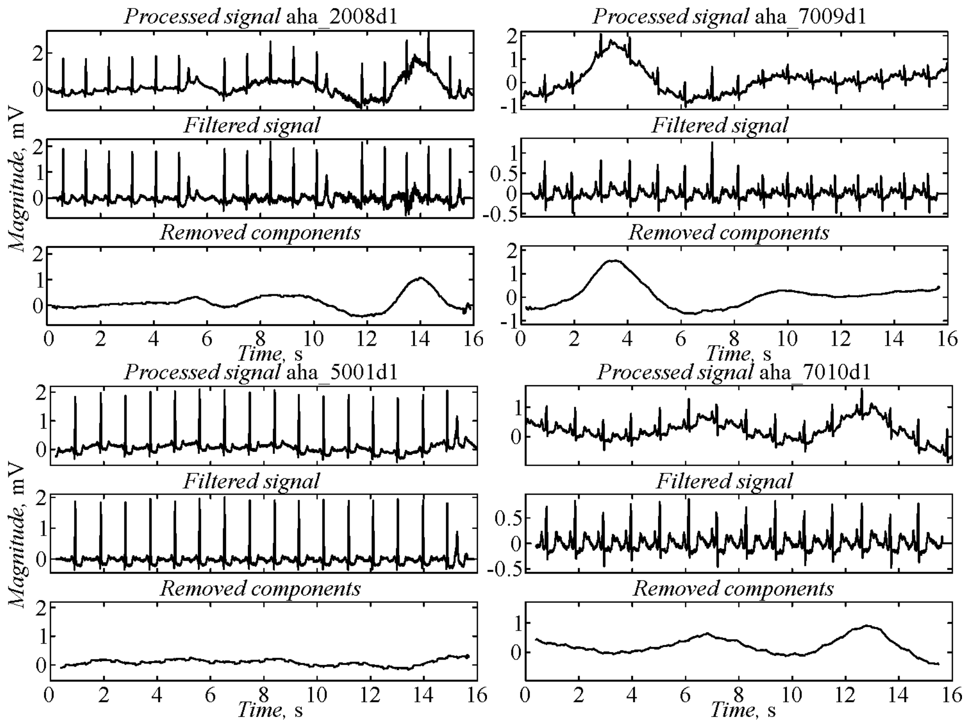

Fig. 10. Results of experiments with real ECG recordings.

6. CONCLUSION

The proposed combined filter for mains interference and baseline drift from ECG signals is based on the methodology for building the so-called “criterion for linearity” (see Fig. 1), the novelty being the pre-summation of d number of filters (1) and the additional convolution (5). The combined filter has a linear phase response and does not introduce phase distortions into the filtered signal. It has a tangent characteristic to the frequency axis, at the mains frequency, which allows it to suppress mains interference even at small frequency deviation. The compiled program for calculating the coefficients of the filter makes it possible to synthesize it for different purities of the mains interference and of the sampling rate, as well as to determine the cut-off frequency of high-pass filtering. The proposed combined filter has a symmetrical impulse response, which allows the number of multiplications when applying it to be twice as small as the number of coefficients. Mains interference suppression of -40 dB can be achieved with a changed mains frequency of no more than 0.05 Hz/s. The reason is the significant length of the filter L and a correspondingly large time interval during which one filtered sample is processed.

As a disadvantage, it can be considered that the harmonics of the mains interference cannot be suppressed. Whole experiments have been performed in a MATLAB environment using 32-bit arithmetic.

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